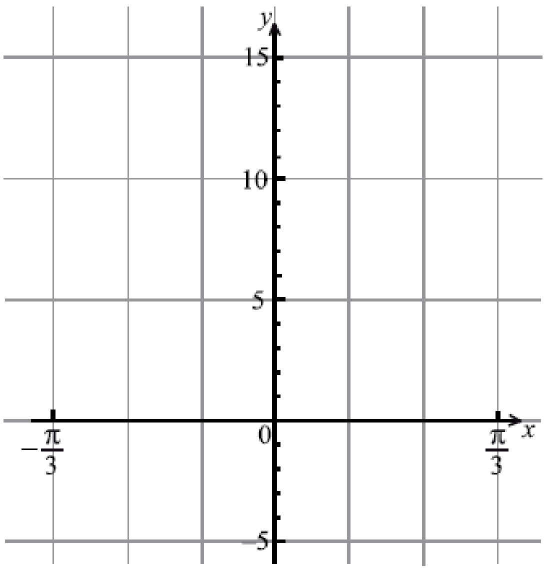
SL Paper 2

Let $f(x) = 4\tan^2 x - 4\sin x$, $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$.

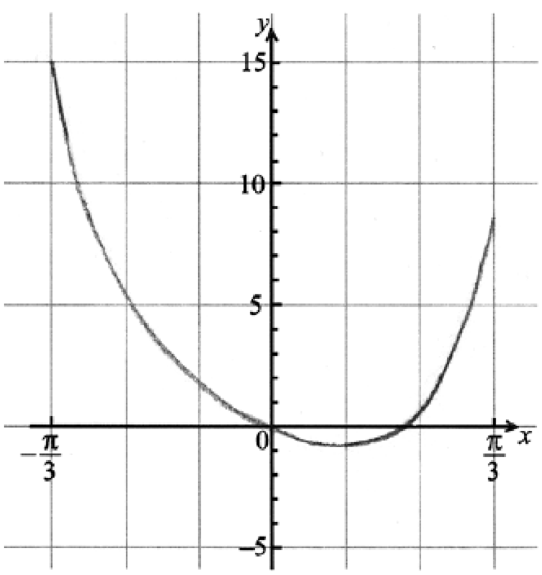
a. On the grid below, sketch the graph of $y = f(x)$. [3]



b. Solve the equation $f(x) = 1$. [3]

Markscheme

a.



A1A1A1 N3

Note: Award ***A1*** for passing through $(0, 0)$, ***A1*** for correct shape, ***A1*** for a range of approximately -1 to 15.

[3 marks]

b. evidence of attempt to solve $f(x) = 1$ ***(M1)*** e.g. line on sketch, using $\tan x = \frac{\sin x}{\cos x}$

$x = -0.207$, $x = 0.772$ ***A1A1*** ***N3***

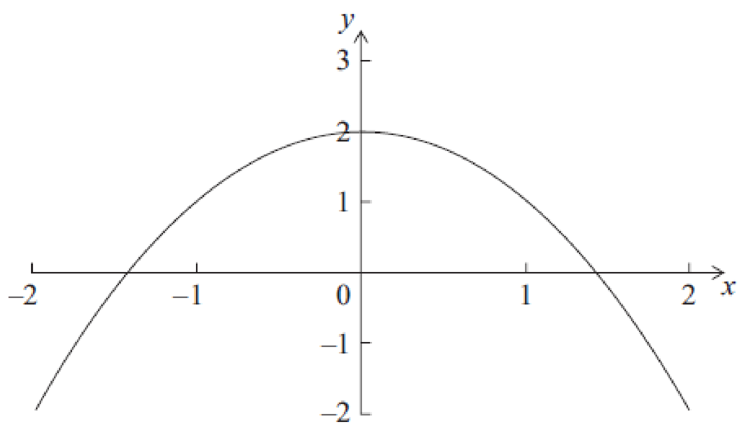
[3 marks]

Examiners report

a. In part (a), some did not realize that they should copy the curve from their GDC, paying attention to domain and range.

b. Not using their GDC, and trying to solve the equation analytically in part (b) proved to be very difficult for many. A common error was to substitute $x = 1$.

Consider $f(x) = 2 - x^2$, for $-2 \le x \le 2$ and $g(x) = \sin e^x$, for $-2 \le x \le 2$. The graph of *f* is given below.



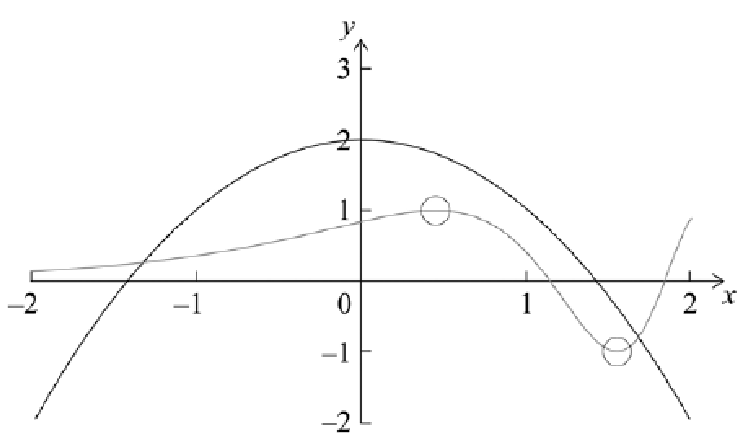
a. On the diagram above, sketch the graph of *g*. [3]

b. Solve $f(x) = g(x)$. [2]

c. Write down the set of values of *x* such that $f(x) > g(x)$. [2]

Markscheme

a.



A1A1A1 ***N3***

[3 marks]

b. $x = -1.32$, $x = 1.68$ (accept $x = -1.41$, $x = 1.39$ if working in degrees)   ***A1A1***   ***N2***

[2 marks]

c. $-1.32 < x < 1.68$ (accept $-1.41 < x < 1.39$ if working in degrees)   ***A2***   ***N2***

[2 marks]

Examiners report

a. This question was answered well by a pleasing number of candidates.

For part (a), many good graphs were seen, though a significant number of candidates either used degrees or a function such as $\sin e^x$. There were students who lost marks for poor diagrams. For example, the shape was correct but the maximum and minimum were not accurate enough.

b. There were candidates who struggled in vain to solve the equation in part (b) algebraically instead of using a GDC. Those that did use their GDCs to solve the equation frequently gave their answers inaccurately, suggesting that they did not know how to use the "zero" function on their calculator but found a rough solution using the "trace" function.

c. In part (c) they often gave the correct solution, or obtained follow-through marks on their incorrect results to part (b).

Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

a. Find $h(x)$. [2]

b. Find $h^{-1}(x)$. [3]

Markscheme

a. attempt to form composite   ***(M1)***

e.g. $f(2x - 5)$

$h(x) = 6x - 15$ ***A1***   ***N2***

[2 marks]

b. interchanging *x* and *y*   ***(M1)***

evidence of correct manipulation   ***(A1)***

e.g. $y + 15 = 6x$, $\frac{x}{6} = y - \frac{5}{2}$

$h^{-1}(x) = \frac{x+15}{6}$ ***A1***   ***N3***

[3 marks]

Examiners report

a. Most candidates handled this question with ease. Some were not familiar with the notation of composite functions assuming that $(f \circ g)(x)$ implied finding the composition and then multiplying this by x . Others misunderstood part (b) and found the reciprocal function or the derivative, indicating they were not familiar with the notation for an inverse function.

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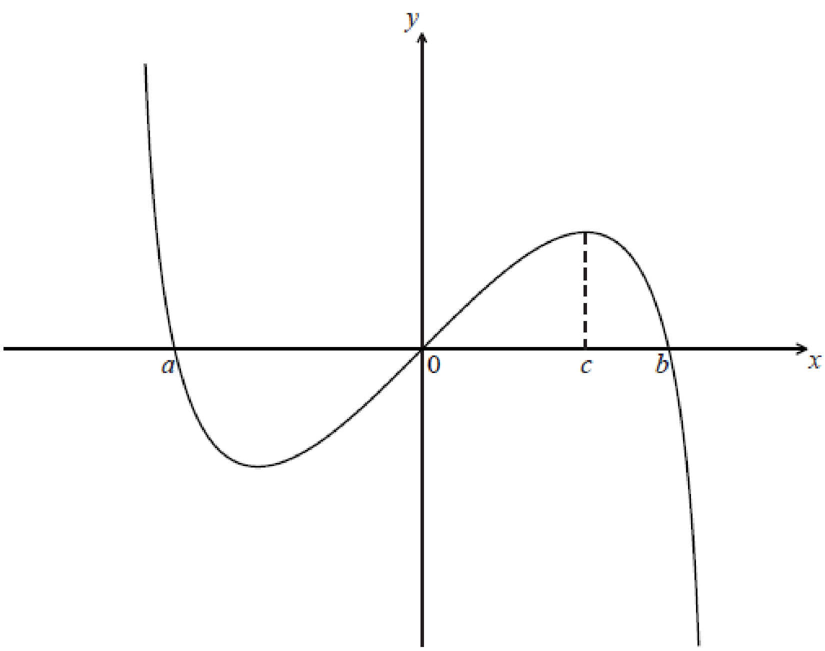
Let $f(x) = x\ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is shown below.



The graph of f crosses the x-axis at $x = a$, $x = 0$ and $x = b$.

a. Find the value of a and of b . [3]

b. The graph of f has a maximum value when $x = c$. [2]

Find the value of c .

c. The region under the graph of f from $x = 0$ to $x = c$ is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

d. Let R be the region enclosed by the curve, the x-axis and the line $x = c$, between $x = a$ and $x = c$. [4]

Find the area of R .

Markscheme

a. evidence of valid approach ***(M1)***

e.g. $f(x) = 0$, graph

$a = -1.73$, $b = 1.73$ $(a = -\sqrt{3}, b = \sqrt{3})$ ***A1A1*** ***N3***

[3 marks]

b. attempt to find max ***(M1)***

e.g. setting $f'(x) = 0$, graph

$c = 1.15$ (accept (1.15, 1.13)) ***A1*** ***N2***

[2 marks]

c. attempt to substitute either limits or the function into formula ***M1***

e.g. $V = \pi\int_0^c [f(x)]^2 \mathrm{d}x$, $\pi\int \left[x \ln(4 - x^2)\right]^2$, $\pi\int_0^{1.149\ldots} y^2 \mathrm{d}x$

$V = 2.16$ ***A2*** ***N2***

[3 marks]

d. valid approach recognizing 2 regions ***(M1)***

e.g. finding 2 areas

correct working ***(A1)***

e.g. $\int_0^{-1.73\ldots} f(x)\mathrm{d}x + \int_0^{1.149\ldots} f(x)\mathrm{d}x$, $-\int_{-1.73\ldots}^0 f(x)\mathrm{d}x + \int_0^{1.149\ldots} f(x)\mathrm{d}x$

area $= 2.07$ (accept 2.06) ***A2*** ***N3***

[4 marks]

Examiners report

a. This question was well done by many candidates. If there were problems, it was often with incorrect or inappropriate GDC use. For example, some candidates used the trace feature to answer parts (a) and (b), which at best, only provides an approximation.

b. This question was well done by many candidates. If there were problems, it was often with incorrect or inappropriate GDC use. For example, some candidates used the trace feature to answer parts (a) and (b), which at best, only provides an approximation.

c. Most candidates were able to set up correct expressions for parts (c) and (d) and if they had used their calculators, could find the correct answers. Some candidates omitted the important parts of the volume formula. Analytical approaches to (c) and (d) were always futile and no marks were gained.

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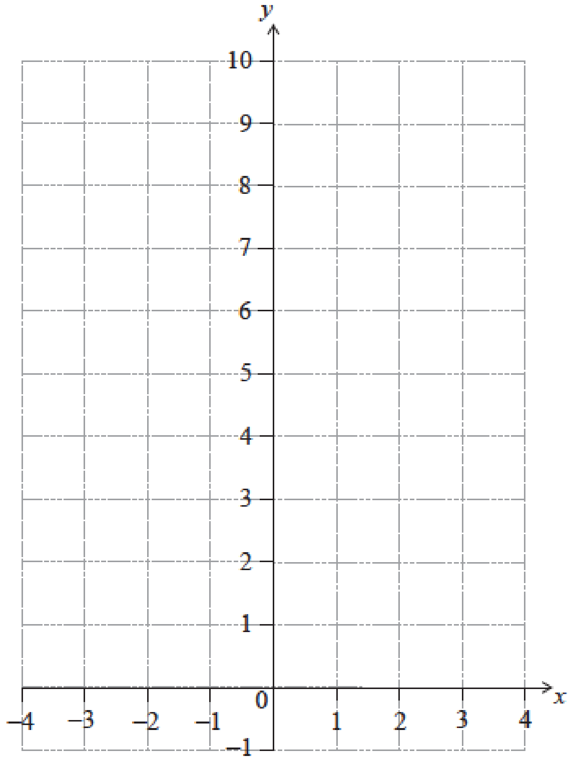
Let $f(x) = \mathrm{e}^{x+1} + 2$, for $-4 \le x \le 1$.

a. On the following grid, sketch the graph of f. [3]

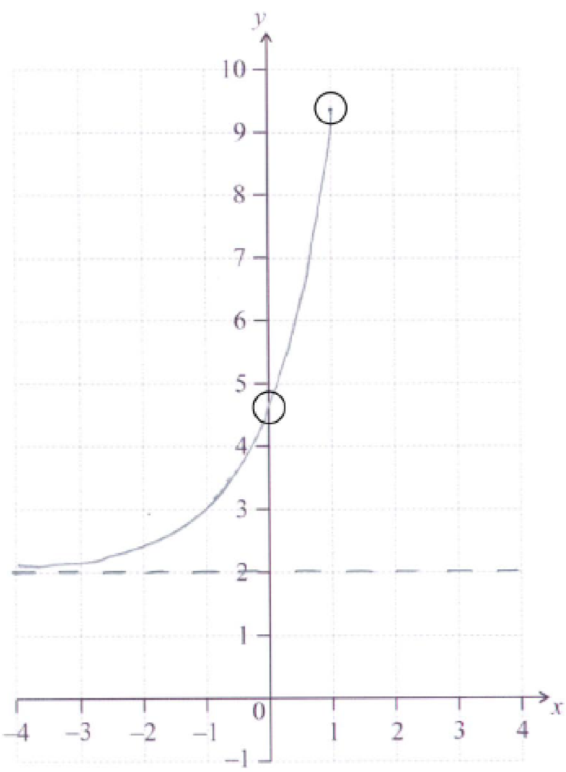


b. The graph of f is translated by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to obtain the graph of a function g. [3]

Find an expression for $g(x)$.

Markscheme

a.



1A1A1 N3

Note: Curve must be approximately correct exponential shape (increasing and concave up). Only if the shape is approximately correct, award the following:

A1 for right end point in circle,

A1 for y-intercept in circle,

A1 for asymptotic to $y = 2$, (must be above $y = 2$).

[3 marks]

b. valid attempt to find g ***(M1)***

eg $f(x-3)-1,\ g(x) = \mathrm{e}^{x+1-3}+2-1,\ \mathrm{e}^{x+1-3},\ 2-1$, sketch

$g(x) = \mathrm{e}^{x-2}+1$ ***A2 N3***

[3 marks]

Total [6 marks]

Examiners report

a. Although this question involved a straightforward use of the GDC, the graphing of this exponential function on a given grid seemed challenging for a number of candidates. Although most candidates were able to graph the correct shape, they did not take into account the domain and range of this function.

Many were inattentive to the asymptotic nature of the function. Very few actually drew the asymptote, which in this case was a relevant feature.

When finding an expression for g, many reversed the direction of one or both of the transformations. The vertical translation was usually correct, but the horizontal shift was poorly done. The most common error was to obtain $g(x) = \mathrm{e}^{x+4}+1$.

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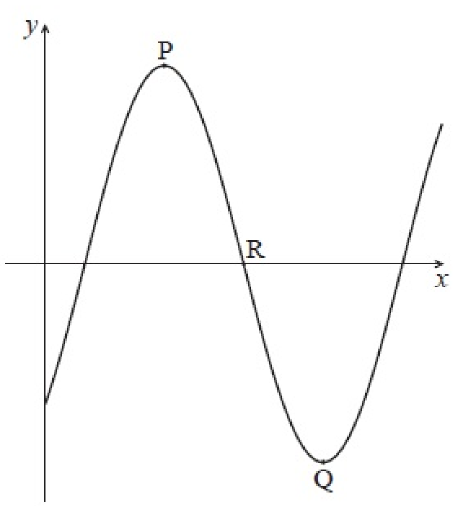
Let $f(x) = a\cos(b(x-c))$. The diagram below shows part of the graph of f , for $0 \le x \le 10$.



The graph has a local maximum at P$(3, 5)$, a local minimum at Q$(7, -5)$, and crosses the x-axis at R.

a(i) and (ii). Write down the value of [2]

(i) a ;

(ii) c .

b. Find the value of b . [2]

c. Find the x-coordinate of R. [2]

Markscheme

a(i) and (ii). (i) $a = 5$ (accept -5) ***A1 N1***

(ii) $c = 3$ (accept $c = 7$, if $a = -5$) ***A1 N1***

Note: Accept other correct values of c, such as 11, -5, etc.

[2 marks]

b. attempt to find period ***(M1)***

e.g. 8 , $b = \frac{2\pi}{\text{period}}$

$0.785398\ldots$

$b = \frac{2\pi}{8}$ (exact), $\frac{\pi}{4}$, 0.785 [0.785, 0.786] (do not accept 45) ***A1 N2***

[2 marks]

c. valid approach ***(M1)***

e.g. $f(x) = 0$, symmetry of curve

$x = 5$ (accept $(5\ ,0)$) ***A1 N2***

[2 marks]

Examiners report

a(i) and (ii). Part (a) (i) was well answered in general. There were more difficulties in finding the correct value of the parameter c.

b. Finding the correct value of b in part (b) also proved difficult as many did not realize the period was equal to 8.

c. Most candidates could handle part (c) without difficulties using their GDC or working with the symmetry of the curve although follow through from errors in part (b) was often not awarded because candidates failed to show any working by writing down the equations they entered into their GDC.

The number of bacteria in two colonies, A and B, starts increasing at the same time.

The number of bacteria in colony A after t hours is modelled by the function $\mathrm{A}(\mathrm{t}) = 12\mathrm{e}^{0.4\mathrm{t}}$.

a. Find the number of bacteria in colony A after four hours. [2]

b. Find the number of bacteria in colony A after four hours. [3]

c. How long does it take for the number of bacteria in colony A to reach 400? [3]

d. The number of bacteria in colony B after t hours is modelled by the function $B(t) = 24\mathrm{e}^{kt}$. [3]

After four hours, there are 60 bacteria in colony B. Find the value of k.

e. The number of bacteria in colony B after t hours is modelled by the function $B(t) = 24\mathrm{e}^{kt}$. [4]

The number of bacteria in colony A first exceeds the number of bacteria in colony B after n hours, where $n \in \mathbb{Z}$. Find the value of n.

Markscheme

a. correct substitution into formula ***(A1)***

eg $12\mathrm{e}^{0.4(0)}$

12 bacteria in the dish ***A1 N2***

[2 marks]

b. correct substitution into formula ***(A1)***

eg $12\mathrm{e}^{0.4(4)}$

59.4363 ***(A1)***

59 bacteria in the dish (integer answer only) ***A1 N3***

[3 marks]

c. correct equation ***(A1)***

eg $A(t) = 400,\ 12\mathrm{e}^{0.4t} = 400$

valid attempt to solve ***(M1)***

eg graph, use of logs

8.76639

8.77 (hours) ***A1 N3***

[3 marks]

d. valid attempt to solve ***(M1)***

eg $n(4) = 60,\ 60 = 24\mathrm{e}^{4k}$, use of logs

correct working ***(A1)***

eg sketch of intersection, $4k = \ln 2.5$

$k = 0.229072$

$k = \frac{\ln 2.5}{4}$ (exact), $k = 0.229$ ***A1 N3***

[3 marks]

e. **METHOD 1**

setting up an equation or inequality (accept any variable for n) ***(M1)***
eg $A(t) > B(t),\ 12\mathrm{e}^{0.4n} = 24\mathrm{e}^{0.229n},\ \mathrm{e}^{0.4n} = 2\mathrm{e}^{0.229n}$
correct working ***(A1)***
eg sketch of intersection, $\mathrm{e}^{0.171n} = 2$
4.05521 (accept 4.05349) ***(A1)***
$n = 5$ (integer answer only) ***A1 N3***
METHOD 2
$A(4) = 59,\ B(4) = 60$ (from earlier work)
$A(5) = 88.668,\ B(5) = 75.446$ ***A1A1***
valid reasoning ***(R1)***
eg $A(4) < B(4)$ **and** $A(5) > B(5)$
$n = 5$ (integer answer only) ***A1 N3***
[4 marks]

Examiners report

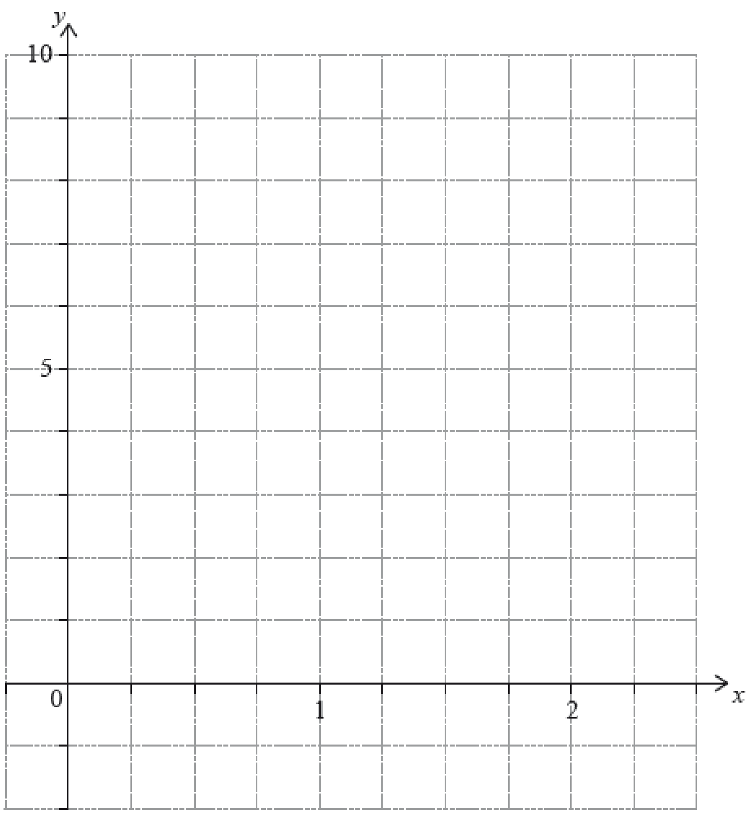
[N/A]

a.
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

Let $f(x) = x^2 - 1$ and $g(x) = x^2 - 2$, for $x \in \mathbb{R}$.

a. Show that $(f \circ g)(x) = x^4 - 4x^2 + 3$. [2]

b. On the following grid, sketch the graph of $(f \circ g)(x)$, for $0 \leqslant x \leqslant 2.25$. [3]



c. The equation $(f \circ g)(x) = k$ has exactly two solutions, for $0 \leqslant x \leqslant 2.25$. Find the possible values of k. [3]

Markscheme

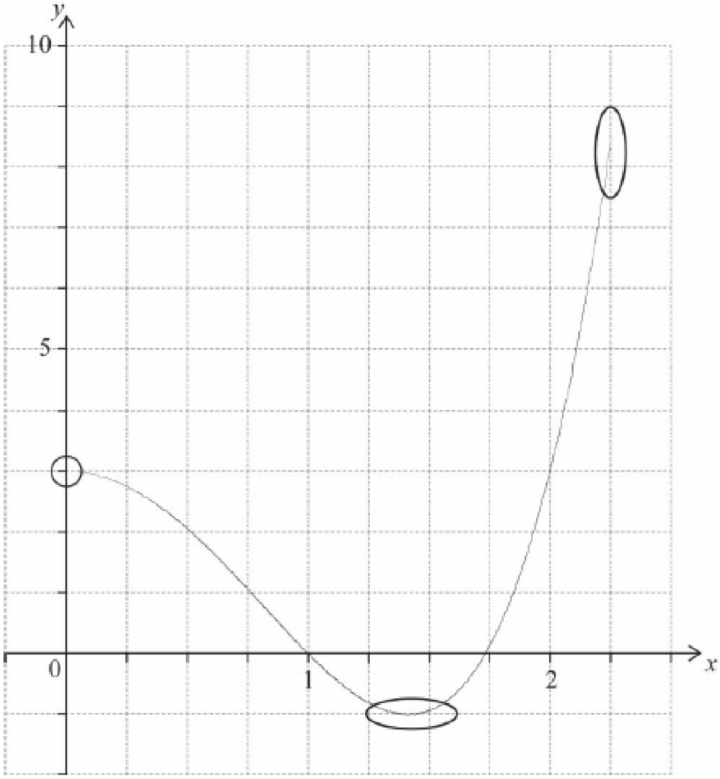
a. attempt to form composite in either order ***(M1)***

eg $f(x^2-2),\ (x^2-1)^2-2$

$(x^4 - 4x^2 + 4) - 1$ ***A1***

$(f \circ g)(x) = x^4 - 4x^2 + 3$ ***AG*** ***N0***

[2 marks]

b. 

A1

A1A1 *N3*

Note: Award ***A1*** for approximately correct shape which changes from concave down to concave up. Only if this ***A1*** is awarded, award the following:

A1 for left hand endpoint in circle **and** right hand endpoint in oval,

A1 for minimum in oval.

[3 marks]

c. evidence of identifying max/min as relevant points ***(M1)***

eg $x = 0,\ 1.41421,\ y = -1,\ 3$

correct interval (inclusion/exclusion of endpoints must be correct) ***A2*** ***N3***

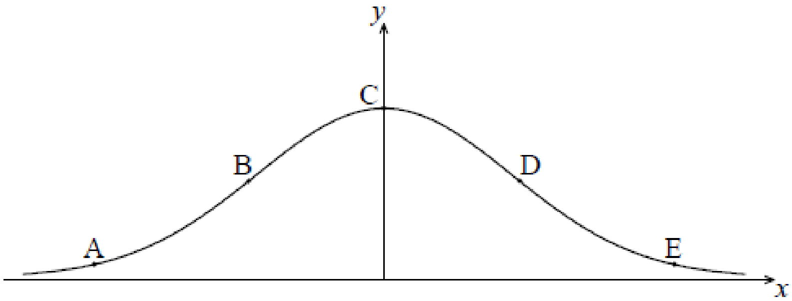
eg $-1 < k \leqslant 3,\ \left]-1,\ 3\right],\ (-1,\ 3]$

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows the graph of $f(x) = \mathrm{e}^{-x^2}$.



The points A, B, C, D and E lie on the graph of f . Two of these are points of inflexion.

a. Identify the **two** points of inflexion. [2]

b(i) and (ii). (i) Find $f'(x)$. [5]

(ii) Show that $f''(x) = (4x^2 - 2)\mathrm{e}^{-x^2}$.

c. Find the x-coordinate of each point of inflexion. [4]

d. Use the second derivative to show that one of these points is a point of inflexion. [4]

Markscheme

a. B, D ***A1A1*** ***N2***

[2 marks]

b(i) and (ii). (i) $f'(x) = -2x\mathrm{e}^{-x^2}$ ***A1A1*** ***N2***

Note: Award ***A1*** for e^{-x^2} and ***A1*** for $-2x$.

(ii) finding the derivative of $-2x$, i.e. -2 ***(A1)***

evidence of choosing the product rule ***(M1)***

e.g. $-2\mathrm{e}^{-x^2} - 2x \times -2x\mathrm{e}^{-x^2}$

$-2\mathrm{e}^{-x^2} + 4x^2\mathrm{e}^{-x^2}$ ***A1***

$f''(x) = (4x^2 - 2)\mathrm{e}^{-x^2}$ ***AG*** ***N0***

[5 marks]

c. valid reasoning ***R1***

e.g. $f''(x) = 0$

attempting to solve the equation ***(M1)***

e.g. $(4x^2 - 2) = 0$, sketch of $f''(x)$

$p = 0.707 \left(= \frac{1}{\sqrt{2}}\right)$, $q = -0.707 \left(= -\frac{1}{\sqrt{2}}\right)$ ***A1A1*** ***N3***

[4 marks]

d. evidence of using second derivative to test values on either side of POI ***M1***

e.g. finding values, reference to graph of f'' , sign table

correct working ***A1A1***

e.g. finding any two correct values either side of POI, checking sign of f'' on either side of POI

reference to sign change of $f''(x)$ ***R1*** ***N0***

[4 marks]

Examiners report

a. Most candidates were able to recognize the points of inflexion in part (a).

b(i) and (ii). Most candidates were able to recognize the points of inflexion in part (a) and had little difficulty with the first and second derivatives in part (b). A few did not recognize the application of the product rule in part (b).

c. Obtaining the *x*-coordinates of the inflexion points in (c) usually did not cause many problems.

d. Only the better-prepared candidates understood how to set up a second derivative test in part (d). Many of those did not show, or clearly indicate, the values of *x* used to test for a point of inflexion, but merely gave an indication of the sign. Some candidates simply resorted to showing that $f''\left(\pm\frac{1}{\sqrt{2}}\right) = 0$, completely missing the point of the question. The necessary condition for a point of inflexion, i.e. $f''(x) = 0$ **and** the change of sign for $f''(x)$, seemed not to be known by the vast majority of candidates.

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a(i) and (ii). Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$. [4]

(i) Find u_4 .

(ii) Find the sum of the infinite sequence.

b(i) and (ii). Consider an arithmetic sequence with *n* terms, with first term (-36) and eighth term (-8) . [5]

(i) Find the common difference.

(ii) Show that $S_n = 2n^2 - 38n$.

c. The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find *n* . [5]

Markscheme

a(i) and (ii). (i) correct approach ***(A1)***

e.g. $u_4 = (40)\frac{1}{2}^{(4-1)}$, listing terms

$u_4 = 5$ ***A1*** ***N2***

(ii) correct substitution into formula for infinite sum ***(A1)***

e.g. $S_\infty = \frac{40}{1-0.5}$, $S_\infty = \frac{40}{0.5}$

$S_\infty = 80$ ***A1*** ***N2***

[4 marks]

b(i) and (ii). (i) attempt to set up expression for u_8 ***(M1)***

e.g. $-36 + (8-1)d$

correct working ***A1***

e.g. $-8 = -36 + (8-1)d$, $\frac{-8-(-36)}{7}$

$d = 4$ ***A1*** ***N2***

(ii) correct substitution into formula for sum ***(A1)***

e.g. $S_n = \frac{n}{2}(2(-36) + (n-1)4)$

correct working ***A1***

e.g. $S_n = \frac{n}{2}(4n - 76)$, $-36n + 2n^2 - 2n$

$S_n = 2n^2 - 38n$ ***AG*** ***N0***

[5 marks]

c. multiplying S_n (AP) by 2 or dividing S (infinite GP) by 2 ***(M1)***

e.g. $2S_n$, $\frac{S_\infty}{2}$, 40

evidence of substituting into $2S_n = S_\infty$ ***A1***

e.g. $2n^2 - 38n = 40$, $4n^2 - 76n - 80 (= 0)$

attempt to solve **their** quadratic (equation) ***(M1)***

e.g. intersection of graphs, formula

$n = 20$ ***A2*** ***N3***

[5 marks]

Examiners report

a(i) and (ii). Most candidates found part (a) straightforward, although a common error in (a)(ii) was to calculate 40 divided by $\frac{1}{2}$ as 20.

b(i) and (ii). In part (b), some candidates had difficulty with the "show that" and worked backwards from the answer given.

c. Most candidates obtained the correct equation in part (c), although some did not reject the negative value of n as impossible in this context.

The following table shows a probability distribution for the random variable X, where $\mathrm{E}(X) = 1.2$.

x	0	1	2	3
$\mathrm{P}(X = x)$	p	$\frac{1}{2}$	$\frac{3}{10}$	q

A bag contains white and blue marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of blue marbles drawn is given by the random variable X.

A game is played in which three marbles are drawn from the bag of ten marbles, without replacement. A player wins a prize if three white marbles are drawn.

a.i. Find q. [2]

a.ii. Find p. [2]

b.i. Write down the probability of drawing three blue marbles. [1]

b.ii. Explain why the probability of drawing three white marbles is $\frac{1}{6}$. [1]

b.iii. The bag contains a total of ten marbles of which w are white. Find w. [3]

d. Grant plays the game until he wins two prizes. Find the probability that he wins his second prize on his eighth attempt. [4]

Markscheme

a.i. correct substitution into $\mathrm{E}(X)$ formula ***(A1)***

eg $0(p)+1(0.5)+2(0.3)+3(q)=1.2$

$q=\frac{1}{30}$, 0.0333 ***A1 N2***

[2 marks]

a.ii. evidence of summing probabilities to 1 ***(M1)***

eg $p+0.5+0.3+q=1$

$p=\frac{1}{6},\ 0.167$ ***A1 N2***

[2 marks]

b.i. $\mathrm{P}\ (3\text{ blue})=\frac{1}{30},\ 0.0333$ ***A1 N1***

[1 mark]

b.ii. valid reasoning ***R1***

eg $\mathrm{P}\ (3\text{ white})=\mathrm{P}(0\text{ blue})$

$\mathrm{P}(3\text{ white})=\frac{1}{6}$ ***AG N0***

[1 mark]

b.iii. valid method ***(M1)***

eg $\mathrm{P}(3\text{ white})=\frac{w}{10}\times\frac{w-1}{9}\times\frac{w-2}{8},\ \frac{{}_wC_3}{{}_{10}C_3}$

correct equation ***A1***

eg $\frac{w}{10}\times\frac{w-1}{9}\times\frac{w-2}{8}=\frac{1}{6},\ \frac{{}_wC_3}{{}_{10}C_3}=0.167$

$w=6$ ***A1 N2***

[3 marks]

d. recognizing one prize in first seven attempts ***(M1)***

eg $\begin{pmatrix}7\\1\end{pmatrix},\ \left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6$

correct working ***(A1)***

eg $\begin{pmatrix}7\\1\end{pmatrix}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6,\ 0.390714$

correct approach ***(A1)***

eg $\binom{7}{1}\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{6}\times\frac{1}{6}$

0.065119

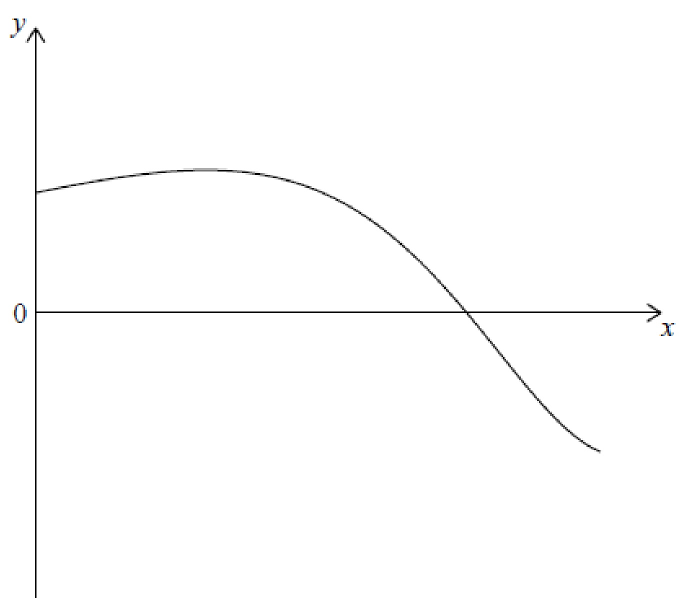
0.0651 ***A1*** ***N2***

[4 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b.i. [N/A]
b.ii. [N/A]
b.iii. [N/A]
d. [N/A]

Let $f(x) = \sin(e^x)$ for $0 \le x \le 1.5$. The following diagram shows the graph of f.



a. Find the *x*-intercept of the graph of f. [2]

b. The region enclosed by the graph of f, the *y*-axis and the *x*-axis is rotated 360° about the *x*-axis. [3]

Find the volume of the solid formed.

Markscheme

a. valid approach ***(M1)***

eg $f(x) = 0,\ \ e^x = 180$ or 0…

1.14472

$x = \ln\pi$ (exact), 1.14 ***A1 N2***

[2 marks]

b. attempt to substitute either their **limits** or the function into formula involving f^2. ***(M1)***

eg $\int_0^{1.14} f^2,\ \pi \int (\sin\ (e^x))^2 dx,\ 0.795135$

2.49799

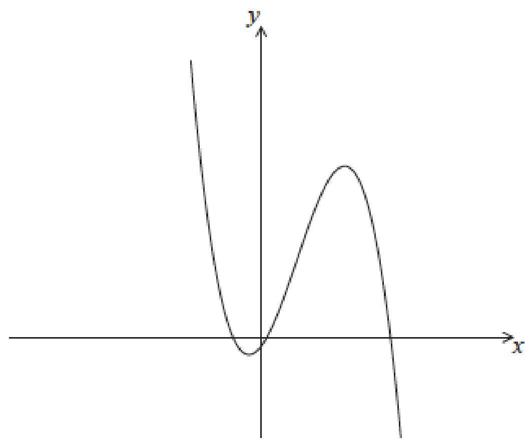
volume = 2.50 ***A2 N3***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

The following diagram shows part of the graph of $f(x) = -2x^3 + 5.1x^2 + 3.6x - 0.4$.



a. Find the coordinates of the local minimum point. [2]

b. The graph of f is translated to the graph of g by the vector $\begin{pmatrix} 0 \\ k \end{pmatrix}$. Find all values of k so that $g(x) = 0$ has exactly one solution. [5]

Markscheme

a. $(-0.3,\ -0.967)$

$x = -0.3$ (exact), $y = -0.967$ (exact) ***A1A1 N2***

[2 marks]

b. y-coordinate of local maximum is $y = 11.2$ ***(A1)***

negating the y-coordinate of one of the max/min ***(M1)***

eg $y = 0.967,\ y = -11.2$

recognizing that the solution set has two intervals ***R1***

eg two answers,

$k < -11.2,\ k > 0.967$ ***A1A1 N3N2***

[5 marks]

Notes: If working shown, do not award the final mark if strict inequalities are not used.

If no working shown, award ***N2*** for $k \le -11.2$ or ***N1*** for $k \ge 0.967$

Total [7 marks]

Examiners report

a. The coordinates of the minimum point was correctly given by most candidates, although some opted for an analytical approach which was often futile and time consuming.

b. In part (b), few students appreciated that the solution set consisted of two **intervals** often giving only one correct interval or equalities. The most common, incorrect approach was an attempt to use the discriminant.

An environmental group records the numbers of coyotes and foxes in a wildlife reserve after t years, starting on 1 January 1995.

Let c be the number of coyotes in the reserve after t years. The following table shows the number of coyotes after t years.

number of years (t)	0	2	10	15	19
number of coyotes (c)	115	197	265	320	406

The relationship between the variables can be modelled by the regression equation $c = at + b$.

a. Find the value of a and of b. [3]

b. Use the regression equation to estimate the number of coyotes in the reserve when $t = 7$. [3]

c. Let f be the number of foxes in the reserve after t years. The number of foxes can be modelled by the equation $f = \frac{2000}{1+99\mathrm{e}^{-kt}}$, where k is a constant. [3]

Find the number of foxes in the reserve on 1 January 1995.

d. After five years, there were 64 foxes in the reserve. Find k. [3]

e. During which year were the number of coyotes the same as the number of foxes? [4]

Markscheme

a. evidence of setup ***(M1)***

eg correct value for a or b

$13.3823,\ 137.482$

$a = 13.4,\ b = 137$ ***A1A1*** ***N3***

[3 marks]

b. correct substitution into **their** regression equation *eg* $13.3823 \times 7 + 137.482$ ***(A1)***

correct calculation

231.158 ***(A1)***

231 (coyotes) (must be an integer) ***A1*** ***N2***

[3 marks]

c. recognizing $t = 0$ ***(M1)***

eg $f(0)$

correct substitution into the model

eg $\frac{2000}{1+99\mathrm{e}^{-k(0)}}$, $\frac{2000}{100}$ ***(A1)***

20 (foxes) ***A1*** ***N2***

[3 marks]

d. recognizing $(5,\ 64)$ satisfies the equation ***(M1)***

eg $f(5) = 64$

correct substitution into the model

eg $64 = \frac{2000}{1+99\mathrm{e}^{-k(5)}}$, $64(1 + 99\backslash(e$^{ - 5k}}) = 2000\) ***(A1)***

0.237124

$k = -\frac{1}{5}\ln\left(\frac{11}{36}\right)$ (exact), 0.237 $[0.237,\ 0.238]$ ***A1*** ***N2***

[3 marks]

e. valid approach ***(M1)***

eg $c = f$, sketch of graphs

correct working ***(A1)***

eg $\frac{2000}{1+99\mathrm{e}^{-0.237124t}} = 13.382t + 137.482$, sketch of graphs, table of values

$t = 12.0403$ ***(A1)***

2007 ***A1*** ***N2***

Note: Exception to the ***FT*** rule. Award ***A1FT*** on their value of t.

[4 marks]

Total [16 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

Let $f'(x) = -24x^3 + 9x^2 + 3x + 1$.

a. There are two points of inflexion on the graph of f . Write down the x-coordinates of these points. [3]

b. Let $g(x) = f''(x)$. Explain why the graph of g has no points of inflexion. [2]

Markscheme

a. valid approach ***R1***

e.g. $f''(x) = 0$, the max and min of f' gives the points of inflexion on f

-0.114, 0.364 (accept $(-0.114, 0.811)$ and $(0.364, 2.13)$) ***A1A1*** ***N1N1***

[3 marks]

b. **METHOD 1**

graph of g is a quadratic function ***R1*** ***N1***

a quadratic function does not have any points of inflexion ***R1*** ***N1***

METHOD 2

graph of g is concave down over entire domain ***R1*** ***N1***

therefore no change in concavity ***R1*** ***N1***

METHOD 3

$g''(x) = -144$ ***R1 N1***

therefore no points of inflexion as $g''(x) \neq 0$ ***R1*** ***N1***

[2 marks]

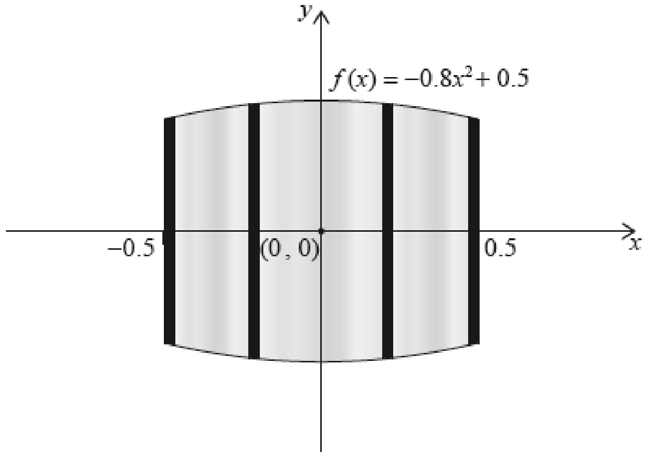
Examiners report

a. There were mixed results in part (a). Students were required to understand the relationships between a function and its derivative and often obtained the correct solutions with incorrect or missing reasoning.

b. In part (b), the question was worth two marks and candidates were required to make two valid points in their explanation. There were many approaches to take here and candidates often confused their reasoning or just kept writing hoping that somewhere along the way they would say something correct to pick up the points. Many confused f' and g' .

All lengths in this question are in metres.

Let $f(x) = -0.8x^2 + 0.5$, for $-0.5 \leqslant x \leqslant 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of f, the x-axis, the line $x = -0.5$ and the line $x = 0.5$ is rotated 360° about the x-axis. This is shown in the following diagram.



a. Use the model to find the volume of the barrel. [3]

b. The empty barrel is being filled with water. The volume V m^3 of water in the barrel after t minutes is given by $V = 0.8(1 - \mathrm{e}^{-0.1t})$. How long will it take for the barrel to be half-full? [3]

Markscheme

a. attempt to substitute correct limits or the function into the formula involving

y^2

eg $\pi \int_{-0.5}^{0.5} y^2 \mathrm{d}x,\ \pi \int (-0.8x^2 + 0.5)^2 \mathrm{d}x$

0.601091

volume $= 0.601\ (\mathrm{m}^3)$ ***A2 N3***

[3 marks]

b. attempt to equate half **their** volume to V ***(M1)***

eg $0.30055 = 0.8(1 - \mathrm{e}^{-0.1t})$, graph

4.71104

4.71 (minutes) ***A2 N3***

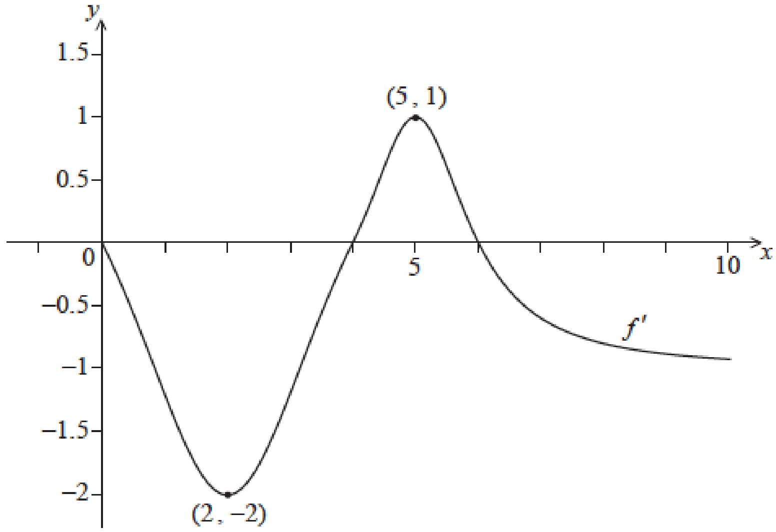
[3 marks]

Examiners report

a. [N/A]

b. [N/A]

Consider a function f, for $0 \leq x \leq 10$. The following diagram shows the graph of f', the derivative of f.



The graph of f' passes through $(2,\ -2)$ and $(5,\ 1)$, and has x-intercepts at 0, 4 and 6.

a. The graph of f has a local maximum point when $x = p$. State the value of p, and justify your answer. [3]

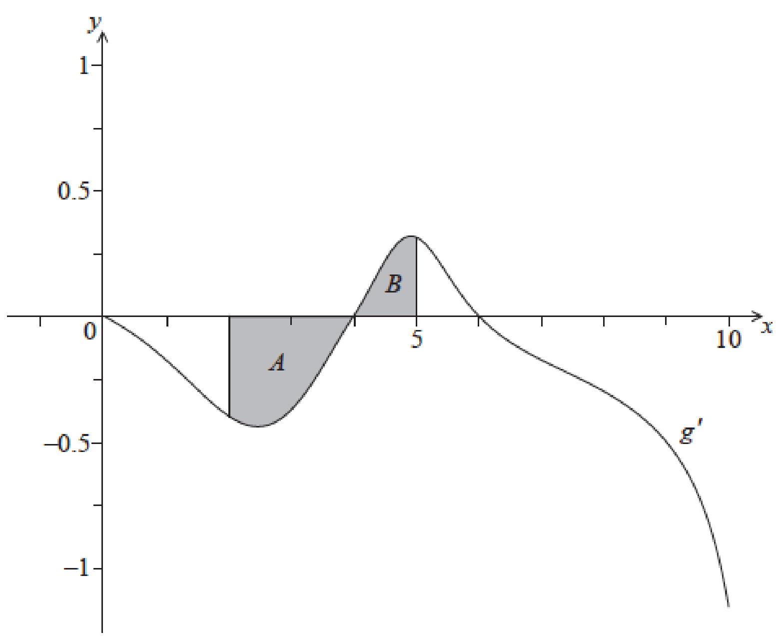
b. Write down $f'(2)$. [1]

c. Let $g(x) = \ln(f(x))$ and $f(2) = 3$. [4]

Find $g'(2)$.

d. Verify that $\ln 3 + \int_2^a g'(x)\mathrm{d}x = g(a)$, where $0 \le a \le 10$. [4]

e. The following diagram shows the graph of g', the derivative of g. [4]



The shaded region A is enclosed by the curve, the x-axis and the line $x = 2$, and has area $0.66\ \text{units}^2$.

The shaded region B is enclosed by the curve, the x-axis and the line $x = 5$, and has area $0.21\ \text{units}^2$.

Find $g(5)$.

Markscheme

a. $p = 6$ ***A1*** ***N1***

recognizing that turning points occur when $f'(x) = 0$ ***R1*** ***N1***

eg correct sign diagram

f' changes from positive to negative at $x = 6$ ***R1*** ***N1***

[3 marks]

b. $f'(2) = -2$ ***A1*** ***N1***

[1 mark]

c. attempt to apply chain rule ***(M1)***

eg $\ln(x)' \times f'(x)$

correct expression for $g'(x)$ ***(A1)***

eg $g'(x) = \frac{1}{f(x)} \times f'(x)$

substituting $x = 2$ into **their** g' ***(M1)***

eg $\frac{f'(2)}{f(2)}$

-0.666667

$g'(2) = -\frac{2}{3}\ (\text{exact}),\ -0.667$ ***A1*** ***N3***

[4 marks]

d. evidence of integrating $g'(x)$ ***(M1)***

eg $g(x)|_2^a,\ g(x)|_a^2$

applying the fundamental theorem of calculus (seen anywhere) ***R1***

eg $\int_2^a g'(x) = g(a) - g(2)$

correct substitution into integral ***(A1)***

eg $\ln 3 + g(a) - g(2),\ \ln 3 + g(a) - \ln(f(2))$

$\ln 3 + g(a) - \ln 3$ ***A1***

$\ln 3 + \int_2^a g'(x) = g(a)$ ***AG*** ***N0***

[4 marks]

e. **METHOD 1**

substituting $a = 5$ into the formula for $g(a)$ ***(M1)***

eg $\int_2^5 g'(x)\mathrm{d}x,\ g(5) = \ln 3 + \int_2^5 g'(x)\mathrm{d}x \quad (\text{do not accept only } g(5))$

attempt to substitute areas ***(M1)***

eg $\ln 3 + 0.66 - 0.21,\ \ln 3 + 0.66 + 0.21$

correct working

eg $g(5) = \ln 3 + (-0.66 + 0.21)$ ***(A1)***

0.648612

$g(5) = \ln 3 - 0.45\ (\text{exact}), 0.649$ ***A1*** ***N3***

METHOD 2

attempt to set up an equation for one shaded region ***(M1)***

eg $\int_4^5 g'(x)\mathrm{d}x = 0.21,\ \int_2^4 g'(x)\mathrm{d}x = -0.66,\ \int_2^5 g'(x)\mathrm{d}x = -0.45$

two correct equations ***(A1)***

eg $g(5) - g(4) = 0.21,\ g(2) - g(4) = 0.66$

combining equations to eliminate $g(4)$ ***(M1)***

eg $g(5) - [\ln 3 - 0.66] = 0.21$

0.648612

$g(5) = \ln 3 - 0.45 \text{ (exact)}, 0.649$ ***A1 N3***

METHOD 3

attempt to set up a definite integral ***(M1)***

eg $\int_2^5 g'(x)\mathrm{d}x = -0.66 + 0.21,\ \int_2^5 g'(x)\mathrm{d}x = -0.45$

correct working ***(A1)***

eg $g(5) - g(2) = -0.45$

correct substitution ***(A1)***

eg $g(5) - \ln 3 = -0.45$

0.648612

$g(5) = \ln 3 - 0.45 \text{ (exact)}, 0.649$ ***A1 N3***

[4 marks]

Total [16 marks]

Examiners report

a. In part (a), many candidates did not get full marks in justifying that $p = 6$ was where the maximum occurs. The derivative changing from positive to negative was not sufficient since there are cases where the derivative changes signs at a value where there is no turning point. Part (c) was very poorly done as most candidates did not recognize the use of the chain rule to find the derivative of $\ln(f(x))$, a fairly basic application for Mathematics SL. In part (d), candidates appeared to have difficulty with the command term “verify”, and even if they were successful, did not make the connection to part (e) where they attempted a variety of interesting ways to find $g(5)$ - the most common approach was to set up two incorrect integrals involving areas A and B. Many students did not realize that integrating a function over an interval where the function is negative gives the opposite of the area between the function and the *x*-axis.

b. In part (a), many candidates did not get full marks in justifying that $p = 6$ was where the maximum occurs. The derivative changing from positive to negative was not sufficient since there are cases where the derivative changes signs at a value where there is no turning point. Part (c) was very poorly done as most candidates did not recognize the use of the chain rule to find the derivative of $\ln(f(x))$, a fairly basic application for Mathematics SL. In part (d), candidates appeared to have difficulty with the command term “verify”, and even if they were successful, did not make the connection to part (e) where they attempted a variety of interesting ways to find $g(5)$ - the most common approach was to set up two incorrect integrals involving areas A and B. Many students did not realize that integrating a function over an interval where the function is negative gives the opposite of the area between the function and the *x*-axis.

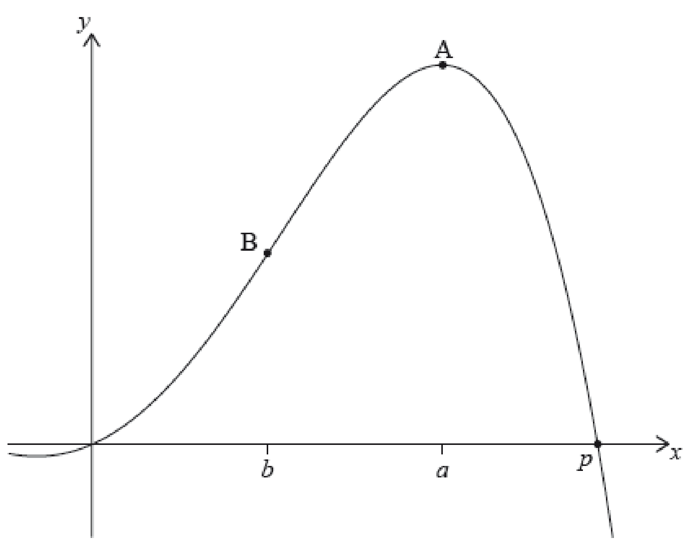
c. In part (a), many candidates did not get full marks in justifying that $p = 6$ was where the maximum occurs. The derivative changing from positive to negative was not sufficient since there are cases where the derivative changes signs at a value where there is no turning point. Part (c) was very poorly done as most candidates did not recognize the use of the chain rule to find the derivative of $\ln(f(x))$, a fairly basic application for Mathematics SL. In part (d), candidates appeared to have difficulty with the command term “verify”, and even if they were successful, did not make

the connection to part (e) where they attempted a variety of interesting ways to find $g(5)$ - the most common approach was to set up two incorrect integrals involving areas A and B. Many students did not realize that integrating a function over an interval where the function is negative gives the opposite of the area between the function and the *x*-axis.

d. In part (a), many candidates did not get full marks in justifying that $p = 6$ was where the maximum occurs. The derivative changing from positive to negative was not sufficient since there are cases where the derivative changes signs at a value where there is no turning point. Part (c) was very poorly done as most candidates did not recognize the use of the chain rule to find the derivative of $\ln(f(x))$, a fairly basic application for Mathematics SL. In part (d), candidates appeared to have difficulty with the command term "verify", and even if they were successful, did not make the connection to part (e) where they attempted a variety of interesting ways to find $g(5)$ - the most common approach was to set up two incorrect integrals involving areas A and B. Many students did not realize that integrating a function over an interval where the function is negative gives the opposite of the area between the function and the *x*-axis.

e. In part (a), many candidates did not get full marks in justifying that $p = 6$ was where the maximum occurs. The derivative changing from positive to negative was not sufficient since there are cases where the derivative changes signs at a value where there is no turning point. Part (c) was very poorly done as most candidates did not recognize the use of the chain rule to find the derivative of $\ln(f(x))$, a fairly basic application for Mathematics SL. In part (d), candidates appeared to have difficulty with the command term "verify", and even if they were successful, did not make the connection to part (e) where they attempted a variety of interesting ways to find $g(5)$ - the most common approach was to set up two incorrect integrals involving areas A and B. Many students did not realize that integrating a function over an interval where the function is negative gives the opposite of the area between the function and the *x*-axis.

Let $f(x) = -0.5x^4 + 3x^2 + 2x$. The following diagram shows part of the graph of f.



There are x-intercepts at $x = 0$ and at $x = p$. There is a maximum at A where $x = a$, and a point of inflexion at B where $x = b$.

a. Find the value of p. [2]

b.i. Write down the coordinates of A. [2]

b.ii. Write down the rate of change of f at A. [1]

c.i. Find the coordinates of B. [4]

c.ii. Find the the rate of change of f at B. [3]

d. Let R be the region enclosed by the graph of f , the x-axis, the line $x = b$ and the line $x = a$. The region R is rotated 360° about the x-axis. [3]

Find the volume of the solid formed.

Markscheme

a. evidence of valid approach ***(M1)***

eg $f(x) = 0,\ y = 0$

2.73205

$p = 2.73$ ***A1 N2***

[2 marks]

b.i. 1.87938, 8.11721

$(1.88,\ 8.12)$ ***A2 N2***

[2 marks]

b.ii. rate of change is 0 (do not accept decimals) ***A1 N1***

[1 marks]

c.i. **METHOD 1 (using GDC)**

valid approach ***M1***

eg $f'' = 0$, max/min on $f',\ x = -1$

sketch of either f' or f'', with max/min or root (respectively) ***(A1)***

$x = 1$ ***A1 N1***

Substituting **their** x value into f ***(M1)***

eg $f(1)$

$y = 4.5$ ***A1 N1***

METHOD 2 (analytical)

$f'' = -6x^2 + 6$ ***A1***

setting $f'' = 0$ ***(M1)***

$x = 1$ ***A1 N1***

substituting **their** x value into f ***(M1)***

eg $f(1)$

$y = 4.5$ ***A1 N1***

[4 marks]

c.ii. recognizing rate of change is f' ***(M1)***

eg $y',\ f'(1)$

rate of change is 6 ***A1 N2***

[3 marks]

d. attempt to substitute either limits or the function into formula ***(M1)***

involving f^2 (accept absence of π and/or $\mathrm{d}x$)

eg $\pi\int\left(-0.5x^4+3x^2+2x\right)^2\mathrm{d}x,\ \int_1^{1.88} f^2$

128.890

$\text{volume} = 129$ ***A2 N3***

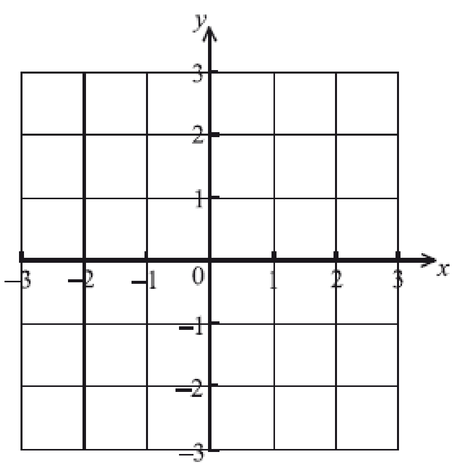
[3 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c.i. [N/A]
c.ii. [N/A]
d. [N/A]

Let $f(x) = x\cos(x - \sin x)$, $0 \le x \le 3$.

a. Sketch the graph of *f* on the following set of axes. [3]

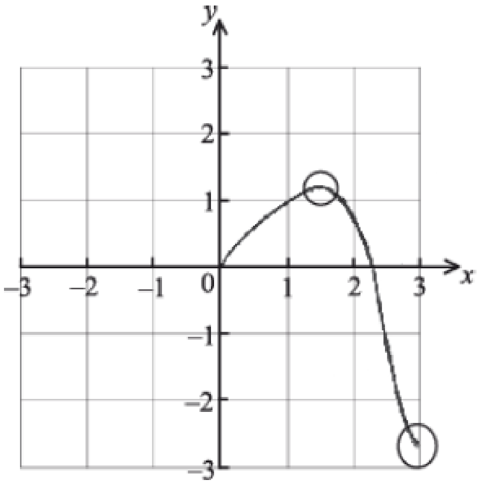


b. The graph of *f* intersects the *x*-axis when $x = a$, $a \ne 0$. Write down the value of *a*. [1]

c. The graph of *f* is revolved 360° about the *x*-axis from $x = 0$ to $x = a$. Find the volume of the solid formed. [4]

Markscheme

a.



A1A2 N3

Notes: Award ***A1*** for correct domain, $0 \leq x \leq 3$. Award ***A2*** for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2.

[3 marks]

b. $a = 2.31$ ***A1 N1***

[1 mark]

c. evidence of using $V = \pi \int [f(x)]^2 \mathrm{d}x$ ***(M1)***

fully correct integral expression ***A2***

e.g. $V = \pi \int_0^{2.31} [x \cos(x - \sin x)]^2 \mathrm{d}x$, $V = \pi \int_0^{2.31} [f(x)]^2 \mathrm{d}x$ ***A1 N2***

$V = 5.90$

[4 marks]

Examiners report

a. Many candidates sketched a clear and smooth freehand curve with the local maximum, x-intercept and endpoints in approximately correct positions. Commonly, candidates sketched a graph across $[-3, 3]$, which neglects the given domain of the function. There were some candidates who sketched a straight line through the origin, presumably from being in the degree mode of their GDC.

b. Many candidates sketched a clear and smooth freehand curve with the local maximum, x-intercept and endpoints in approximately correct positions. Commonly, candidates sketched a graph across $[-3, 3]$, which neglects the given domain of the function. There were some candidates who sketched a straight line through the origin, presumably from being in the degree mode of their GDC.

c. A good number of candidates could set up the correct integral expression for volume, but surprisingly few were able to use their GDC to find the correct value. Some attempted to analytically integrate the square of this unusual function, expending valuable time in this effort. A small but significant number of candidates wrote a final answer as 1.88π , which accrued the accuracy penalty.

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 \mathrm{e}^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

a. (i) Find the value of k. [3]

(ii) Interpret the meaning of the value of k.

b. Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$. [5]

Markscheme

a. (i) valid approach ***(M1)***

eg $0.9 = \mathrm{e}^{k(1)}$

$k = -0.105360$

$k = \ln 0.9\ (\text{exact}),\ -0.105$ ***A1 N2***

(ii) correct interpretation ***R1 N1***

eg population is decreasing, growth rate is negative

[3 marks]

b. **METHOD 1**

valid approach (accept an equality, but do not accept 0.74) ***(M1)***

eg $P < 0.75P_0,\ P_0\mathrm{e}^{kt} < 0.75P_0,\ 0.75 = \mathrm{e}^{t\ln 0.9}$

valid approach to solve **their** inequality ***(M1)***

eg logs, graph

$t > 2.73045\ (\text{accept } t = 2.73045)\ (2.73982 \text{ from } -0.105)$ ***A1***

28 years ***A2 N2***

METHOD 2

valid approach which gives both crossover values accurate to at least 2 sf ***A2***

eg $\frac{P_{2.7}}{P_0} = 0.75241\ldots,\ \frac{P_{2.8}}{P_0} = 0.74452\ldots$

$t = 2.8$ ***(A1)***

28 years ***A2 N2***

[5 marks]

Examiners report

a. Part (a) was generally done well, with many candidates able to find the value of k correctly and to interpret its meaning. Lack of accuracy was occasionally a concern, with some candidates writing their value of k to 2 significant figures or evaluating $\ln(0.9)$ incorrectly.

b. Few candidates were successful in part (b) with many unable to set up an inequality or equation which would allow them to find the condition on t. Some were able to find the value of t in decades but most were unable to correctly interpret their inequality in terms of the least number of whole years. While a solution through analytic methods was readily available, very few students attempted to use their GDC to solve their initial equation or inequality.

The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y dollars	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

On 1 January 2010, Lina buys a car which has travelled $11\,000$ km.

The price of a car decreases by 5% each year.

Lina will sell her car when its price reaches $10\,000$ dollars.

a. (i) Find the correlation coefficient. [4]

(ii) Write down the value of a and of b.

b. Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. [3]

c. Calculate the price of Lina's car after 6 years. [4]

d. Find the year when Lina sells her car. [4]

Markscheme

a. **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

(i) valid approach ***(M1)***

eg correct value for r (or for a or b seen in (ii))

-0.994347

$r = -0.994$ ***A1 N2***

(ii) $-1.58095,\ 33480.3$

$a = -1.58,\ b = 33500$ ***A1A1 N2***

[4 marks]

b. **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

correct substitution into **their** regression equation

eg $-1.58095(11000)\ + 33480.3$ ***(A1)***

$16\,089.85\ (16\,120 \text{ from 3sf})$ ***(A1)***

$\text{price} = 16\,100\ (\text{dollars})$ (must be rounded to the nearest 100 dollars) ***A1 N3***

[3 marks]

c. **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

METHOD 1

valid approach ***(M1)***

eg $P \times (\text{rate})^t$

$\text{rate} = 0.95$ (may be seen in their expression) ***(A1)***

correct expression ***(A1)***

eg 16100×0.95^6

$11\,834.97$

$11\,800$ (dollars) ***A1 N2***

METHOD 2

attempt to find all six terms ***(M1)***

eg $(((16\,100 \times 0.95) \times 0.95) \ldots) \times 0.95$, table of values

5 correct values (accept values that round correctly to the nearest dollar)

$15\,295, \; 14\,530, \; 13\,804, \; 13\,114, \; 12\,458$ ***A2***

$11\,835$

$11\,800$ (dollars) ***A1 N2***

[4 marks]

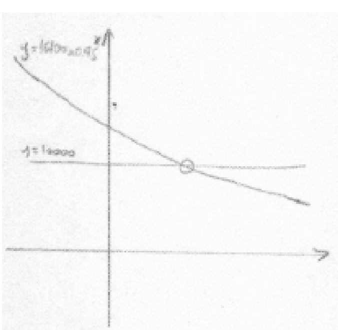
d. **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

METHOD 1

correct equation ***(A1)***

eg $16\,100 \times 0.95^x = 10\,000$

valid attempt to solve ***(M1)***

eg  , using logs

9.28453 ***(A1)***

year 2019 ***A1 N2***

METHOD 2

valid approach using table of values ***(M1)***

both crossover values (accept values that round correctly to the nearest dollar) ***A2***

eg $\text{P} = 10\,147 \; (1 \text{ Jan } 2019), \; \text{P} = 9\,639.7 \; (1 \text{ Jan } 2020)$

year 2019 ***A1 N2***

[4 marks]

Examiners report

a. Although the question talked about the regression equation, a few students tried to find the values of *a* and *b* by forming two equations with the coordinates of two points from the table. A considerable number of candidates did not write the value of the correlation coefficient or gave an incorrect one. It can be that a GDC feature (Diagnostics) from some calculators was turned off.

b. Part (b) was generally well done, with many candidates earning follow through marks. There were some difficulties in rounding the answer to the nearest 100 dollars.

c. Part (c) was attempted in two different ways: recognizing the correct rate 0.95 and then finding the price of the car after 6 years. Some of these candidates used a formula similar to the one for terms of a geometric sequence, $P \times (\text{rate})^{t-1}$, but substituted t by 6 and hence, got an incorrect result.

Others listed all six values to obtain the answer. When using this method, the problem was using less accurate intermediate results and hence, not getting the first 5 correct values of the car.

Many candidates either missed out questions 8 (c) and (d) or multiplied either $0.05 \times 6 \times 16\,100$ or $0.95 \times 6 \times 16\,100$ and failed to notice that the answer did not make sense. Other students tried to use the sum formula for a geometric series.

d. Many candidates either missed out questions 8 (c) and (d) or multiplied either $0.05 \times 6 \times 16\,100$ or $0.95 \times 6 \times 16\,100$ and failed to notice that the answer did not make sense. Other students tried to use the sum formula for a geometric series.

Part (d) was attempted using a graphical approach as well as analytically using logarithms to find the year in which Lina would sell the car, though many failed in giving the correct year. Common answers were “in the ninth year” or “in 2020”. The same happened to those candidates who used a table of values and found the price of the car after 9 years and 10 years. These candidates should be reminded to show both “crossover” values for a table method to be valid.

Let $f(x) = \frac{2x-6}{1-x}$, for $x \neq 1$.

a. For the graph of f [5]

(i) find the x-intercept;

(ii) write down the equation of the vertical asymptote;

(iii) find the equation of the horizontal asymptote.

b. Find $\lim\limits_{x \to \infty} f(x)$. [2]

Markscheme

a. (i) valid approach ***(M1)***

eg sketch, $f(x) = 0,\ 0 = 2x - 6$

$x = 3$ or $(3,\ 0)$ ***A1 N2***

(ii) $x = 1$ (must be equation) ***A1 N1***

(iii) valid approach ***(M1)***

eg sketch, $\frac{2x}{-1x}$, inputting large values of x, L'Hopital's rule

$y = -2$ (must be equation) ***A1*** ***N2***

[5 marks]

b. valid approach ***(M1)***

eg recognizing that $\lim\limits_{x \to \infty}$ is related to the horizontal asymptote, table with large values of x, their y value from (a)(iii), L'Hopital's rule

$\lim\limits_{x \to \infty} f(x) = -2$ ***A1*** ***N2***

[2 marks]

Total [7 marks]

Examiners report

a. Part (a) was generally well done with candidates using both algebraic and graphical approaches to obtain solutions. There are still some who do not identify their asymptotes using equations.

b. Candidates rarely appreciated the relevance of the horizontal asymptote in (b), and often attempted a long, and often unsuccessful, algebraic approach to find the limit.

The number of bacteria, n , in a dish, after t minutes is given by $n = 800\mathrm{e}^{0.13t}$.

a. Find the value of n when $t = 0$. [2]

b. Find the rate at which n is increasing when $t = 15$. [2]

c. After k minutes, the rate of increase in n is greater than 10000 bacteria per minute. Find the least value of k , where $k \in \mathbb{Z}$. [4]

Markscheme

a. $n = 800\mathrm{e}^0$ ***(A1)***

$n = 800$ ***A1*** ***N2***

[2 marks]

b. evidence of using the derivative ***(M1)***

$n'(15) = 731$ ***A1*** ***N2***

[2 marks]

c. **METHOD 1**

setting up inequality (accept equation or reverse inequality) ***A1***

e.g. $n'(t) > 10000$

evidence of appropriate approach ***M1***

e.g. sketch, finding derivative

$k = 35.1226\ldots$ ***(A1)***

least value of k is 36 ***A1*** ***N2***

METHOD 2

$n'(35) = 9842$, **and** $n'(36) = 11208$ ***A2***

least value of k is 36 ***A2*** ***N2***

[4 marks]

Examiners report

a. This question seemed to be challenging for the great majority of the candidates.

Part (a) was generally well answered.

b. This question seemed to be challenging for the great majority of the candidates.

Part (a) was generally well answered but in parts (b) and (c) they did not consider that rates of change meant they needed to use differentiation. Most students completely missed or did not understand that the question was asking about the instantaneous rate of change, which resulted in the fact that most of them used the original equation. Some did attempt to find an average rate of change over the time interval, but even fewer attempted to use the derivative.

Of those who did realize to use the derivative in (b), a vast majority calculated it by hand instead of using their GDC feature to evaluate it.

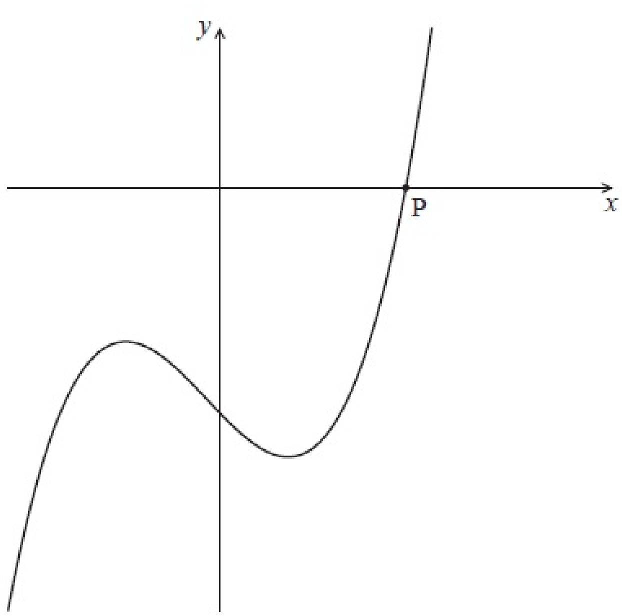
c. This question seemed to be challenging for the great majority of the candidates.

Part (a) was generally well answered but in parts (b) and (c) they did not consider that rates of change meant they needed to use differentiation. Most students completely missed or did not understand that the question was asking about the instantaneous rate of change, which resulted in the fact that most of them used the original equation. Some did attempt to find an average rate of change over the time interval, but even fewer attempted to use the derivative.

Of those who did realize to use the derivative in (b), a vast majority calculated it by hand instead of using their GDC feature to evaluate it.

The inequality for part (c) was sometimes well solved using the original function but many failed to round their answers to the nearest integer.

Let $f(x) = x^3 - 2x - 4$. The following diagram shows part of the curve of f .



The curve crosses the x-axis at the point P.

a. Write down the x-coordinate of P. [1]

b. Write down the gradient of the curve at P. [2]

c. Find the equation of the normal to the curve at P, giving your equation in the form $y = ax + b$. [3]

Markscheme

a. $x = 2$ (accept $(2, 0)$) ***A1*** ***N1***

[1 mark]

b. evidence of finding gradient of f at $x = 2$ ***(M1)***

e.g. $f'(2)$

the gradient is 10 ***A1*** ***N2***

[2 marks]

c. evidence of negative reciprocal of gradient ***(M1)***

e.g. $\frac{-1}{f'(x)}$, $-\frac{1}{10}$

evidence of correct substitution into equation of a line ***(A1)***

e.g. $y - 0 = \frac{-1}{10}(x - 2)$, $0 = -0.1(2) + b$

$y = -\frac{1}{10}x + \frac{2}{10}$ (accept $a = -0.1$, $b = 0.2$) ***A1*** ***N2***

[3 marks]

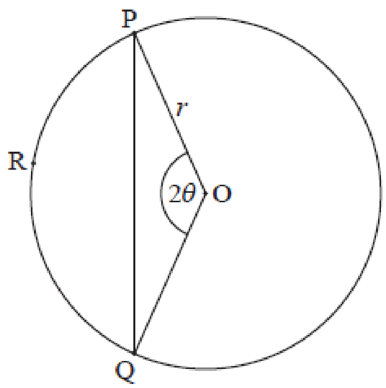
Examiners report

a. This question was generally done well.

b. Most candidates did not use their GDC in part (b), resulting in a variety of careless errors occasionally arising either in differentiating or substituting.

c. There were some candidates who did not know the relationship between gradients of perpendicular lines while others found the equation of the tangent rather than the normal in part (c).

Consider the following circle with centre O and radius r .



The points P, R and Q are on the circumference, $\mathrm{P\widehat{O}Q} = 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

a. Use the cosine rule to show that $\mathrm{PQ} = 2r\sin\theta$. [4]

b. Let l be the length of the arc PRQ . [5]

Given that $1.3\mathrm{PQ} - l = 0$, find the value of θ .

c(i) and (ii). Consider the function $f(\theta) = 2.6\sin\theta - 2\theta$, for $0 < \theta < \frac{\pi}{2}$. [4]

(i) Sketch the graph of f .

(ii) Write down the root of $f(\theta) = 0$.

d. Use the graph of f to find the values of θ for which $l < 1.3\mathrm{PQ}$. [3]

Markscheme

a. correct substitution into cosine rule ***A1***

e.g. $\mathrm{PQ}^2 = r^2 + r^2 - 2(r)(r)\cos(2\theta)$, $\mathrm{PQ}^2 = 2r^2 - 2r^2(\cos(2\theta))$

substituting $1 - 2\sin^2\theta$ for $\cos 2\theta$ (seen anywhere) ***A1***

e.g. $\mathrm{PQ}^2 = 2r^2 - 2r^2(1 - 2\sin^2\theta)$

working towards answer ***(A1)***

e.g. $\mathrm{PQ}^2 = 2r^2 - 2r^2 + 4r^2\sin^2\theta$

recognizing $2r^2 - 2r^2 = 0$ (including crossing out) (seen anywhere)

e.g. $\mathrm{PQ}^2 = 4r^2\sin^2\theta$, $\mathrm{PQ} = \sqrt{4r^2\sin^2\theta}$

$\mathrm{PQ} = 2r\sin\theta$ ***AG*** ***N0***

[4 marks]

b. $\mathrm{PRQ} = r \times 2\theta$ (seen anywhere) ***(A1)***

correct set up ***A1***

e.g. $1.3 \times 2r\sin\theta - r \times (2\theta) = 0$

attempt to eliminate r ***(M1)***

correct equation in terms of the one variable θ ***(A1)***

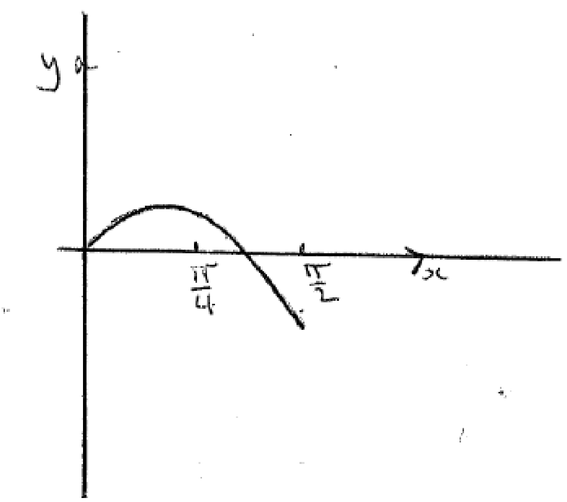
e.g. $1.3 \times 2\sin\theta - 2\theta = 0$

1.221496215

$\theta = 1.22$ (accept 70.0° (69.9)) ***A1*** ***N3***

[5 marks]

c(i) and (ii).



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape, ***A1*** for x-intercept in approximately correct position, ***A1*** for domain. Do not penalise if sketch starts at origin.

(ii) 1.221496215

$\theta = 1.22$ ***A1*** ***N1***

[4 marks]

d. evidence of appropriate approach (may be seen earlier) ***M2***

e.g. $2\theta < 2.6 \sin \theta$, $0 < f(\theta)$, showing positive part of sketch

$0 < \theta < 1.221496215$

$0 < \theta = 1.22$ (accept $\theta < 1.22$) ***A1*** ***N1***

[3 marks]

Examiners report

a. This exercise seemed to be challenging for the great majority of the candidates, in particular parts (b), (c) and (d).

Part (a) was generally attempted using the cosine rule, but many failed to substitute correctly into the right hand side or skipped important steps. A high percentage could not arrive at the given expression due to a lack of knowledge of trigonometric identities or making algebraic errors, and tried to force their way to the given answer.

The most common errors included taking the square root too soon, and sign errors when distributing the negative after substituting $\cos 2\theta$ by $1 - 2\sin^2\theta$.

b. This exercise seemed to be challenging for the great majority of the candidates, in particular parts (b), (c) and (d).

In part (b), most candidates understood what was required but could not find the correct length of the arc PRQ mainly due to substituting the angle by θ instead of 2θ .

c(i) and (ii). Regarding part (c), many valid approaches were seen for the graph of *f*, making a good use of their GDC. A common error was finding a second or third solution outside the domain. A considerable amount of sketches were missing a scale.

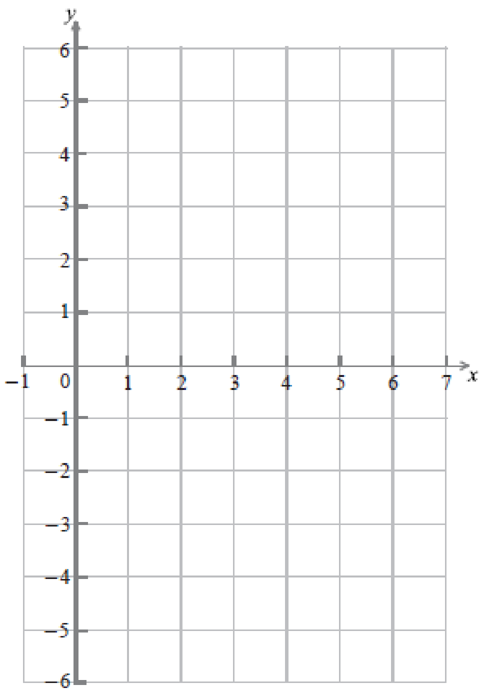
There were candidates who achieved the correct equation but failed to realize they could use their GDC to solve it.

d. Part (d) was attempted by very few, and of those who achieved the correct answer not many were able to show the method they used.

Let $f(x) = x \cos x$, for $0 \le x \le 6$.

a. Find $f'(x)$. [3]

b. On the grid below, sketch the graph of $y = f'(x)$. [4]



Markscheme

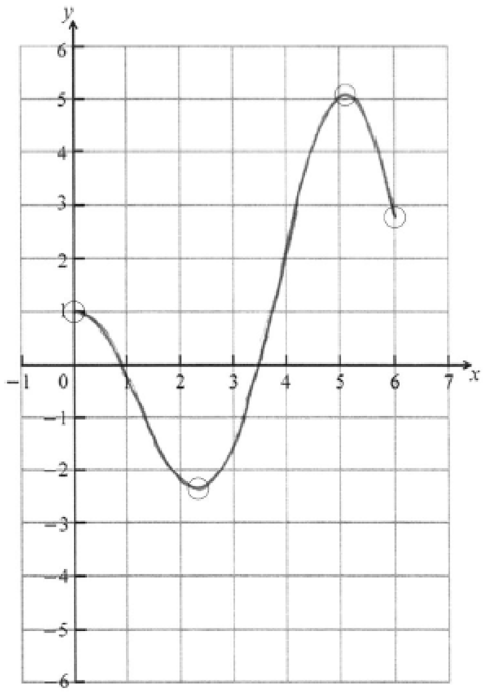
a. evidence of choosing the product rule ***(M1)***

e.g. $x \times (-\sin x) + 1 \times \cos x$

$f'(x) = \cos x - x \sin x$ ***A1A1*** ***N3***

[3 marks]

b.



A1A1A1A1 ***N4***

Note: Award ***A1*** for correct domain, $0 \le x \le 6$ with endpoints in circles, ***A1*** for approximately correct shape, ***A1*** for local minimum in circle, ***A1*** for local maximum in circle.

[4 marks]

Examiners report

a. This problem was well done by most candidates. There were some candidates that struggled to apply the product rule in part (a) and often wrote nonsense like $-x\sin x = -\sin x^2$.

b. In part (b), few candidates were able to sketch the function within the required domain and a large number of candidates did not have their calculator in the correct mode.

The quadratic equation $kx^2 + (k-3)x + 1 = 0$ has two equal real roots.

a. Find the possible values of k. [5]

b. **Write down** the values of k for which $x^2 + (k-3)x + k = 0$ has two equal real roots. [2]

Markscheme

a. attempt to use discriminant ***(M1)***

correct substitution, $(k-3)^2 - 4 \times k \times 1$ ***(A1)***

setting **their** discriminant equal to zero ***M1***

e.g. $(k-3)^2 - 4 \times k \times 1 = 0$, $k^2 - 10k + 9 = 0$

$k = 1$, $k = 9$ ***A1A1*** ***N3***

[5 marks]

b. $k = 1$, $k = 9$ ***A2*** ***N2***

[2 marks]

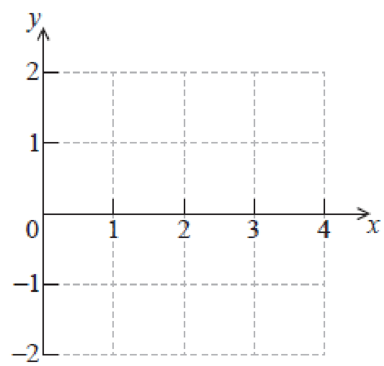
Examiners report

a. Although some candidates correctly considered the discriminant to find the possible values of k , many of them did not set it equal to 0, writing an inequality instead.

b. In part (b), some students realized that the discriminants in parts (a) and (b) were the same, earning follow through marks just by writing the same (often incorrect) answers they got in part (a). Many, however, did not see the connection between the two parts.

Let $g(x) = \frac{1}{2}x \sin x$, for $0 \le x \le 4$.

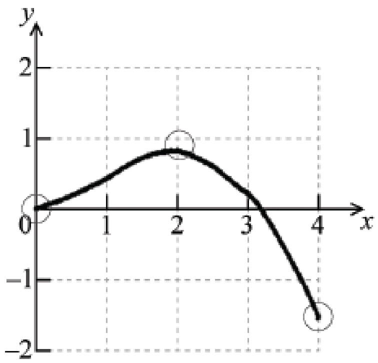
a. Sketch the graph of g on the following set of axes. [4]



b. Hence find the value of x for which $g(x) = -1$. [2]

Markscheme

a.



A1A1A1A1 *N4*

Note: Award *A1* for approximately correct shape, *A1* for left end point in circle, *A1* for local maximum in circle, *A1* for right end point in circle.

[4 marks]

b. attempting to solve $g(x) = -1$ *(M1)*

e.g. marking coordinate on graph, $\frac{1}{2}x\sin x + 1 = 0$

$x = 3.71$ *A1* *N2*

[2 marks]

Examiners report

a. This question was well done by the majority of candidates. Most sketched an approximately correct shape in the given domain, though some candidates did not realize they had to set their GDC to radians, producing a meaningless sketch. Candidates need to be aware that unless otherwise specified, questions will expect radians to be used. The most confident candidates used a table to aid their graphing. Although most recognized the need of the GDC to answer part (b), some used the trace function, hence obtaining an inaccurate result, while others attempted a fruitless analytical approach. Merely stating "using GDC" is insufficient evidence of method; a sketch or an equation set equal to zero are both examples of appropriate evidence.

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Let $f(x) = (x-1)(x-4)$.

a. Find the x-intercepts of the graph of f. [3]

b. The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. [3]

Find the volume of the solid formed.

Markscheme

a. valid approach ***(M1)***

eg $f(x) = 0$, sketch of parabola showing two x-intercepts

$x = 1,\ x = 4$ (accept $(1, 0), (4, 0)$) ***A1A1 N3***

[3 marks]

b. attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\int_1^4 (f(x))^2 \mathrm{d}x,\ \pi \int ((x-1)(x-4))^2$

volume $= 8.1\pi$ (exact), 25.4 ***A2 N3***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$ for $0 \le x \le 9$.

a. On the same diagram, sketch the graphs of *f* and *g* . [3]

b. Consider the graph of f . Write down [4]

(i) the *x*-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude.

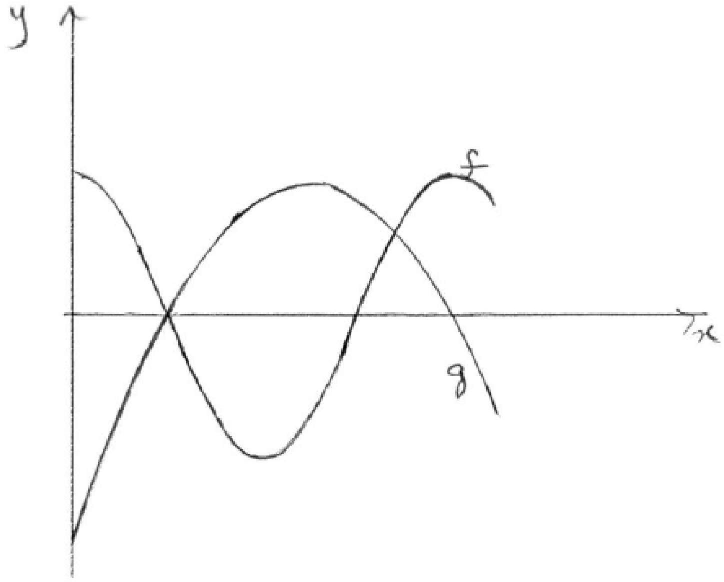
c. Consider the graph of *g* . Write down [3]

(i) the two *x*-intercepts;

(ii) the equation of the axis of symmetry.

d. Let *R* be the region enclosed by the graphs of *f* and *g* . Find the area of *R*. [5]

Markscheme

a. 

A1A1A1 *N3*

Note: Award *A1* for *f* being of sinusoidal shape, with 2 maxima and one minimum, *A1* for *g* being a parabola opening down, *A1* for **two** intersection points in approximately correct position.

[3 marks]

b. (i) $(2, 0)$ (accept $x = 2$) *A1* *N1*

(ii) period $= 8$ *A2* *N2*

(iii) amplitude $= 5$ *A1* *N1*

[4 marks]

c. (i) $(2, 0)$, $(8, 0)$ (accept $x = 2$, $x = 8$) *A1A1* *N1N1*

(ii) $x = 5$ (must be an equation) *A1* *N1*

[3 marks]

d. **METHOD 1**

intersect when $x = 2$ and $x = 6.79$ (may be seen as limits of integration) *A1A1*

evidence of approach *(M1)*

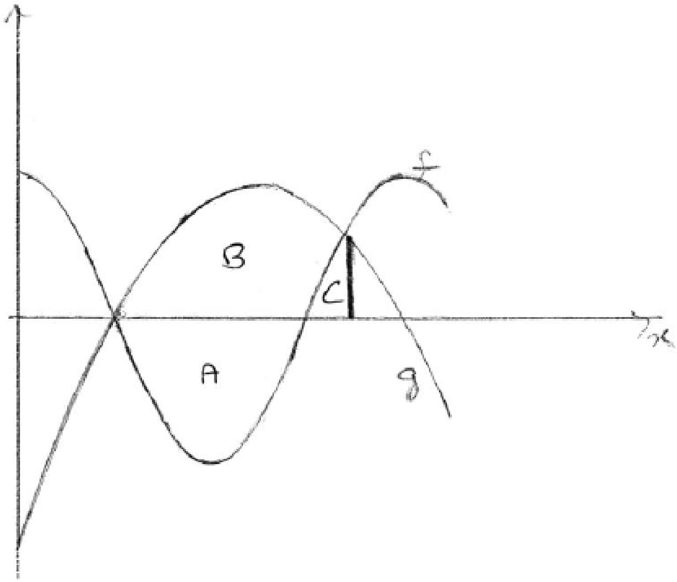
e.g. $\int g - f$, $\int f(x)\mathrm{d}x - \int g(x)\mathrm{d}x$, $\int_2^{6.79} \left((-0.5x^2 + 5x - 8) - \left(5\cos\frac{\pi}{4}x\right)\right)$

area $= 27.6$ *A2* *N3*

METHOD 2

intersect when $x = 2$ and $x = 6.79$ (seen anywhere) *A1A1*

evidence of approach using a sketch of g and f , or $g - f$. *(M1)*



e.g. area = $A + B - C$, $12.7324 + 16.0938 - 1.18129\ldots$

area $= 27.6$ ***A2*** ***N3***

[5 marks]

Examiners report

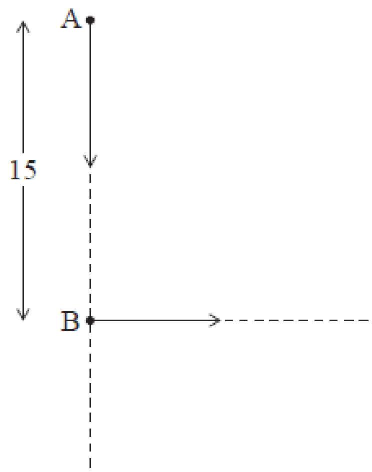
a. Graph sketches were much improved over previous sessions. Most candidates graphed the two functions correctly, but many ignored the domain restrictions.

b. Many candidates found parts (b) and (c) accessible, although quite a few did not know how to find the period of the cosine function.

c. Many candidates found parts (b) and (c) accessible, although quite a few did not know how to find the period of the cosine function.

d. Part (d) proved elusive to many candidates. Some used creative approaches that split the area into parts above and below the *x*-axis; while this leads to a correct result, few were able to achieve it. Many candidates were unable to use their GDCs effectively to find points of intersection and the subsequent area.

The following diagram shows two ships A and B. At noon, ship A was 15 km due north of ship B. Ship A was moving south at 15 km h^{-1} and ship B was moving east at 11 km h^{-1}.



a(i) and (ii). Find the distance between the ships [5]

(i) at 13:00;

(ii) at 14:00.

b. Let $s(t)$ be the distance between the ships *t* hours after noon, for $0 \le t \le 4$. [6]

Show that $s(t) = \sqrt{346t^2 - 450t + 225}$.

c. Sketch the graph of $s(t)$. [3]

d. Due to poor weather, the captain of ship A can only see another ship if they are less than 8 km apart. Explain why the captain cannot see ship B between noon and 16:00. [3]

Markscheme

a(i) and (ii). (i) evidence of valid approach ***(M1)***

e.g. ship A where B was, B 11 km away

distance $= 11$ ***A1 N2***

(ii) evidence of valid approach ***(M1)***

e.g. new diagram, Pythagoras, vectors

$s = \sqrt{15^2 + 22^2}$ ***(A1)***

$\sqrt{709} = 26.62705$

$s = 26.6$ ***A1 N2***

Note: Award ***M0A0A0*** for using the formula given in part (b).

[5 marks]

b. evidence of valid approach ***(M1)***

e.g. a table, diagram, formula $d = r \times t$

distance ship A travels *t* hours after noon is $15(t-1)$ ***(A2)***

distance ship B travels in *t* hours after noon is $11t$ ***(A1)***

evidence of valid approach ***M1***

e.g. $s(t) = \sqrt{[15(t-1)]^2 + (11t)^2}$

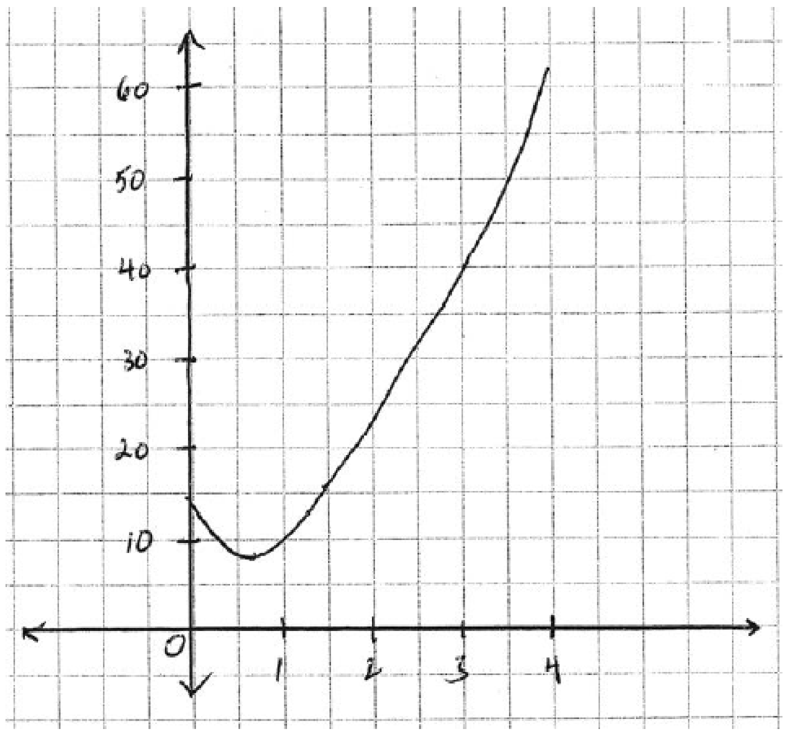
correct simplification ***A1***

e.g. $\sqrt{225(t^2 - 2t + 1) + 121t^2}$

$s(t) = \sqrt{346t^2 - 450t + 225}$ ***AG*** ***N0***

[6 marks]

c.



A1A1A1 ***N3***

Note: Award ***A1*** for shape, ***A1*** for minimum at approximately $(0.7, 9)$, ***A1*** for domain.

[3 marks]

d. evidence of valid approach ***(M1)***

e.g. $s'(t) = 0$, find minimum of $s(t)$, graph, reference to "more than 8 km"

$\min = 8.870455\ldots$ (accept 2 or more sf) ***A1***

since $s_{\min} > 8$, captain cannot see ship B ***R1*** ***N0***

[3 marks]

Examiners report

a(i) and (ii). Part (a) was generally well done although some candidates incorrectly used the function given in part (b) to find the required values. There was evidence that some candidates are not comfortable with a 24-hour clock.

b. Candidates had difficulty generalizing the problem and therefore, were unable to show how the function $s(t)$ was obtained in part (b).

c. Surprisingly, the graph in part (c) was not well done. Candidates often ignored the given domain, provided no indication of scale, and drew "V" shapes or parabolas.

d. In part (d), candidates simply regurgitated the question without providing any significant evidence for their statements that the two ships must have been more than 8 km apart.

Let $f(x) = \frac{1}{x-1} + 2$, for $x > 1$.

Let $g(x) = ae^{-x} + b$, for $x \geqslant 1$. The graphs of f and g have the same horizontal asymptote.

a. Write down the equation of the horizontal asymptote of the graph of f. [2]

b. Find $f'(x)$. [2]

c. Write down the value of b. [2]

d. Given that $g'(1) = -e$, find the value of a. [4]

e. There is a value of x, for $1 < x < 4$, for which the graphs of f and g have the same gradient. Find this gradient. [4]

Markscheme

a. $y = 2$ (correct equation only) ***A2 N2***

[2 marks]

b. valid approach ***(M1)***

eg $(x-1)^{-1} + 2,\ f'(x) = \frac{0(x-1)-1}{(x-1)^2}$

$-(x-1)^{-2},\ f'(x) = \frac{-1}{(x-1)^2}$ ***A1 N2***

[2 marks]

c. correct equation for the asymptote of g

eg $y = b$ ***(A1)***

$b = 2$ ***A1 N2***

[2 marks]

d. correct derivative of *g* (seen anywhere) ***(A2)***

eg $g'(x) = -a\mathrm{e}^{-x}$

correct equation ***(A1)***

eg $-\mathrm{e} = -a\mathrm{e}^{-1}$

7.38905

$a = \mathrm{e}^2\ (\text{exact}),\ 7.39$ ***A1 N2***

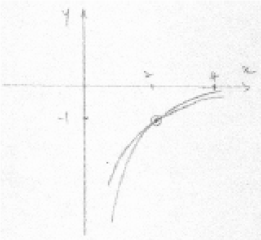
[4 marks]

e. attempt to equate **their** derivatives ***(M1)***

eg $f'(x) = g'(x),\ \frac{-1}{(x-1)^2} = -a\mathrm{e}^{-x}$

valid attempt to solve **their** equation ***(M1)***

eg correct value outside the domain of f such as 0.522 or 4.51,



correct solution (may be seen in sketch) ***(A1)***

eg $x = 2,\ (2,\ -1)$

gradient is -1 ***A1*** ***N3***

[4 marks]

Examiners report

a. Part (a) was in general well answered. Many candidates lost the marks for writing 2 or $y \neq 2$ instead of $y = 2$.

b. In part (b) some candidates got confused and found $f^{-1}(x)$ instead of $f'(x)$. When calculating the derivative, two types of approaches were seen. Most of the ones who rewrote the function as $f(x) = (x-1)^{-1} + 2$, applied the chain rule correctly. Those who tried to apply the quotient rule made various mistakes: incorrect derivative of a constant, incorrect multiplication by zero, wrong subtraction order in the numerator, omitted the negative sign in the answer.

c. In (c), most candidates were coherent and obtained the same value as the one written in part (a).

d. In part (d) many candidates did not manage to differentiate the function g correctly. Of those who could, the equation was generally well solved algebraically.

e. For part (e), not many candidates wrote a correct equation with their derivatives. There was mixed performance for this question, as those who knew they needed to use their GDC managed to obtain an answer, while many got tangled in unsuccessful attempts to solve the equation algebraically. Many candidates tried to solve quite complex equations ‘manually’ instead of trying to graph the expressions on their calculators and finding the value of x at the point of intersection. Of those students who tried to solve graphically only a small percentage actually sketched the two curves that they were considering. This sketch is particularly useful to examiners to see how the student is thinking, or what steps s/he is taking to solve the equations.

Only a few realized that the question asked for the gradient, which was represented by the y-coordinate of the point of intersection, rather than the x-coordinate.

Let $f(t) = 2t^2 + 7$, where $t > 0$. The function *v* is obtained when the graph of *f* is transformed by

a stretch by a scale factor of $\frac{1}{3}$ parallel to the *y*-axis,

followed by a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

a. Find $v(t)$, giving your answer in the form $a(t-b)^2 + c$. [4]

b. A particle moves along a straight line so that its velocity in ms^{-1} , at time t seconds, is given by v . Find the distance the particle travels between $t = 5.0$ and $t = 6.8$. [3]

Markscheme

a. applies vertical stretch parallel to the y-axis factor of $\frac{1}{3}$ ***(M1)***

e.g. multiply by $\frac{1}{3}$, $\frac{1}{3}f(t)$, $\frac{1}{3} \times 2$

applies horizontal shift 2 units to the right ***(M1)***

e.g. $f(t-2)$, $t-2$

applies a vertical shift 4 units down ***(M1)***

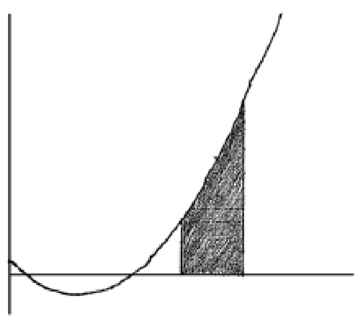
e.g. subtracting 4, $f(t) - 4$, $\frac{7}{3} - 4$

$v(t) = \frac{2}{3}(t-2)^2 - \frac{5}{3}$ ***A1*** ***N4***

[4 marks]

b. recognizing that distance travelled is area under the curve ***M1***

e.g. $\int v$, $\frac{2}{9}(t-2)^3 - \frac{5}{3}t$, sketch



distance = 15.576 (accept 15.6) ***A2*** ***N2***

[3 marks]

Examiners report

a. While a number of candidates had an understanding of each transformation, most had difficulty applying them in the correct order, and few obtained the completely correct answer in part (a). Many earned method marks for discerning three distinct transformations. Few candidates knew to integrate to find the distance travelled. Many instead substituted time values into the velocity function or its derivative and subtracted. A number of those who did recognize the need for integration attempted an analytic approach rather than using the GDC, which often proved unsuccessful.

b. While a number of candidates had an understanding of each transformation, most had difficulty applying them in the correct order, and few obtained the completely correct answer in part (a). Many earned method marks for discerning three distinct transformations. Few candidates knew to integrate to find the distance travelled. Many instead substituted time values into the velocity function or its derivative and subtracted. A number of those who did recognize the need for integration attempted an analytic approach rather than using the GDC, which often proved unsuccessful.

Jose takes medication. After t minutes, the concentration of medication left in his bloodstream is given by $A(t) = 10(0.5)^{0.014t}$, where A is in milligrams per litre.

a. Write down $A(0)$. [1]

b. Find the concentration of medication left in his bloodstream after 50 minutes. [2]

c. At 13:00, when there is no medication in Jose's bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again? [5]

Markscheme

a. $A(0) = 10$ ***A1 N1***

[1 mark]

b. substitution into formula ***(A1)***

e.g. $10(0.5)^{0.014(50)}$, $A(50)$

$A(50) = 6.16$ ***A1 N2***

[2 marks]

c. set up equation ***(M1)***

e.g. $A(t) = 0.395$

attempting to solve ***(M1)***

e.g. graph, use of logs

correct working ***(A1)***

e.g. sketch of intersection, $0.014t \log 0.5 = \log 0.0395$

$t = 333.00025 \ldots$ ***A1***

correct time 18:33 or 18:34 (accept 6:33 or 6:34 but nothing else) ***A1 N3***

[5 marks]

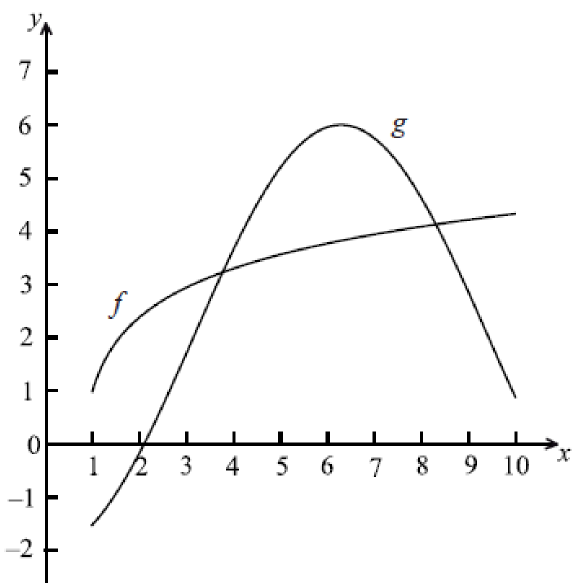
Examiners report

a. For a later question in Section A, a pleasing number of candidates made good progress. Some candidates believed that raising a base to the zero power gave zero which indicated that they most likely did not begin by analysing the function with their GDC. For part (c), many candidates could set up the equation correctly and had some idea to apply logarithms but became lost in the algebra. Those who used their GDC to find when the function equalled 0.395 typically did so successfully. A common error for those who obtained a correct value for time in minutes was to treat 5.55 hours as 5 hours and 55 minutes after 13:00.

b. For a later question in Section A, a pleasing number of candidates made good progress. Some candidates believed that raising a base to the zero power gave zero which indicated that they most likely did not begin by analysing the function with their GDC. For part (c), many candidates could set up the equation correctly and had some idea to apply logarithms but became lost in the algebra. Those who used their GDC to find when the function equalled 0.395 typically did so successfully. A common error for those who obtained a correct value for time in minutes was to treat 5.55 hours as 5 hours and 55 minutes after 13:00.

c. For a later question in Section A, a pleasing number of candidates made good progress. Some candidates believed that raising a base to the zero power gave zero which indicated that they most likely did not begin by analysing the function with their GDC. For part (c), many candidates could set up the equation correctly and had some idea to apply logarithms but became lost in the algebra. Those who used their GDC to find when the function equalled 0.395 typically did so successfully. A common error for those who obtained a correct value for time in minutes was to treat 5.55 hours as 5 hours and 55 minutes after 13:00.

The following diagram shows the graphs of $f(x) = \ln(3x - 2) + 1$ and $g(x) = -4\cos(0.5x) + 2$, for $1 \le x \le 10$.



a(i) and (ii). Let A be the area of the region **enclosed** by the curves of f and g. [6]

(i) Find an expression for A.

(ii) Calculate the value of A.

b(i) and (ii). (i) Find $f'(x)$. [4]

(ii) Find $g'(x)$.

c. There are two values of x for which the gradient of f is equal to the gradient of g. Find both these values of x. [4]

Markscheme

a(i) and (ii). (i) intersection points $x = 3.77$, $x = 8.30$ (may be seen as the limits) ***(A1)(A1)*** approach involving subtraction and integrals ***(M1)***

fully correct expression ***A2***

e.g. $\int_{3.77}^{8.30}((-4\cos(0.5x)+2)-(\ln(3x-2)+1))\mathrm{d}x$, $\int_{3.77}^{8.30} g(x)\mathrm{d}x - \int_{3.77}^{8.30} f(x)\mathrm{d}x$

(ii) $A = 6.46$ ***A1*** ***N1***

[6 marks]

b(i) and (ii). (i) $f'(x) = \frac{3}{3x-2}$ ***A1A1*** ***N2***

Note: Award ***A1*** for numerator (3), ***A1*** for denominator $(3x-2)$, but penalize 1 mark for additional terms.

(ii) $g'(x) = 2\sin(0.5x)$ ***A1A1*** ***N2***

Note: Award ***A1*** for 2, ***A1*** for $\sin(0.5x)$, but penalize 1 mark for additional terms.

[4 marks]

c. evidence of using derivatives for gradients ***(M1)***

correct approach ***(A1)***

e.g. $f'(x) = g'(x)$, points of intersection

$x = 1.43$, $x = 6.10$ ***A1A1*** ***N2N2***

[4 marks]

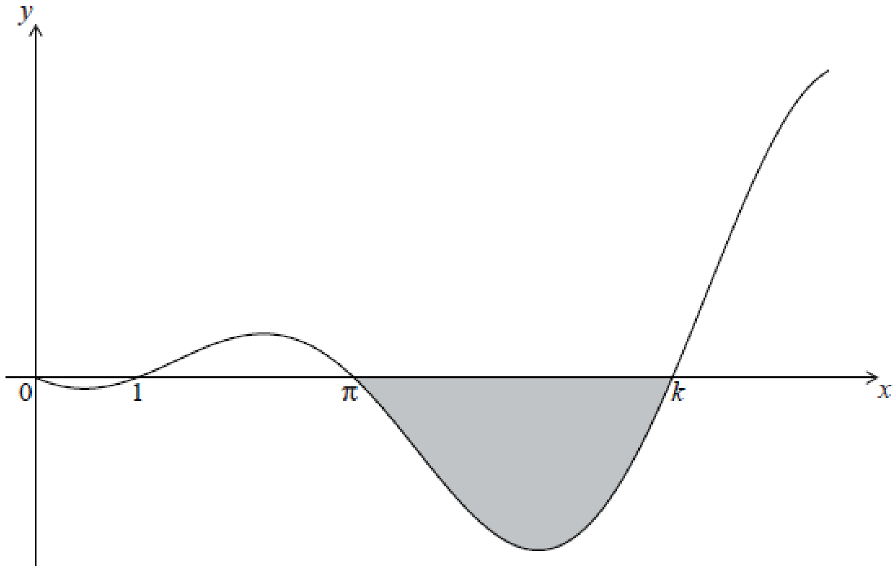
Examiners report

a(i) and (ii). Many candidates did not make good use of the GDC in this problem. Most had the correct expression but incorrect limits. Some tried to integrate to find the area without using their GDC. This became extremely complicated and time consuming.

b(i) and (ii). In part (b), the chain rule was not used by some.

c. Most candidates realized the relationship between the gradient and the first derivative and set the two derivatives equal to one another. Once again many did not realize that the intersection could be easily found on their GDC.

The graph of $y = (x-1)\sin x$, for $0 \le x \le \frac{5\pi}{2}$, is shown below.



The graph has x-intercepts at 0, 1, π and k .

a. Find k . [2]

b. The shaded region is rotated 360° about the x-axis. Let V be the volume of the solid formed. [3]

Write down an expression for V .

c. The shaded region is rotated 360° about the x-axis. Let V be the volume of the solid formed. [2]

Find V .

Markscheme

a. evidence of valid approach ***(M1)***

e.g. $y = 0$, $\sin x = 0$

$2\pi = 6.283185\ldots$

$k = 6.28$ ***A1 N2***

[2 marks]

b. attempt to substitute either limits or the function into formula ***(M1)***

(accept absence of $\mathrm{d}x$)

e.g. $V = \pi \int_\pi^k (f(x))^2 \mathrm{d}x$, $\pi \int ((x-1)\sin x)^2$, $\pi \int_\pi^{6.28\ldots} y^2 \mathrm{d}x$

correct expression ***A2 N3***

e.g. $\pi \int_\pi^{6.28} (x-1)^2 \sin^2 x \mathrm{d}x$, $\pi \int_\pi^{2\pi} ((x-1)\sin x)^2 \mathrm{d}x$

[3 marks]

c. $V = 69.60192562\ldots$

$V = 69.6$ ***A2 N2***

[2 marks]

Examiners report

a. Candidates showed marked improvement in writing fully correct expressions for a volume of revolution. Common errors of course included the omission of dx , using the given domain as the upper and lower bounds of integration, forgetting to square their function and/or the omission of π . There were still many who were unable to use their calculator successfully to find the required volume.

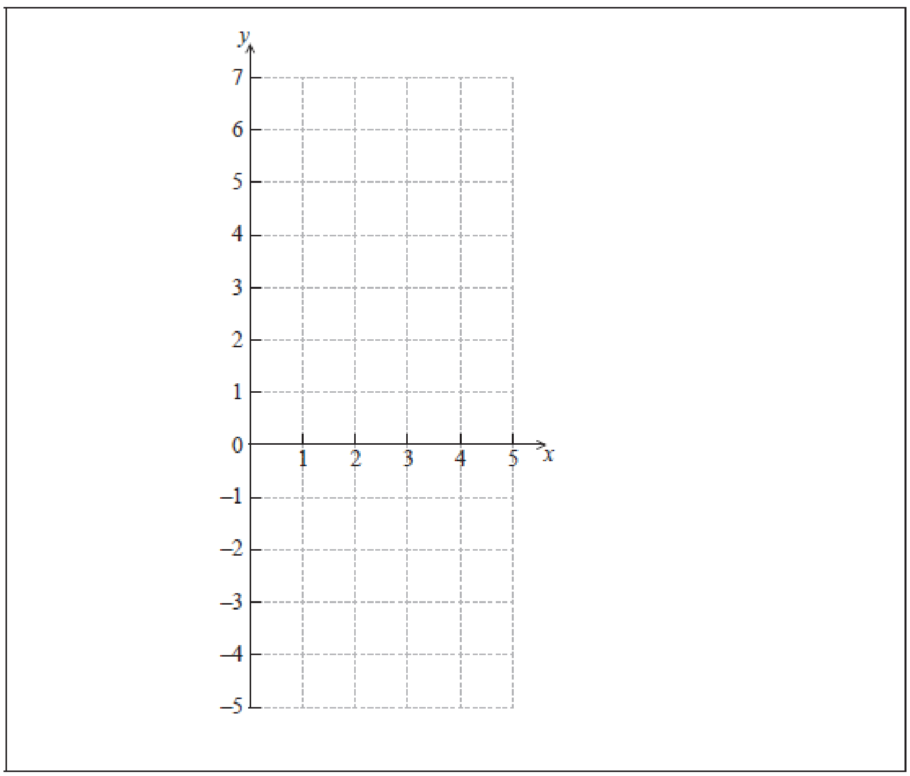
b. Candidates showed marked improvement in writing fully correct expressions for a volume of revolution. Common errors of course included the omission of dx , using the given domain as the upper and lower bounds of integration, forgetting to square their function and/or the omission of π . There were still many who were unable to use their calculator successfully to find the required volume.

c. Candidates showed marked improvement in writing fully correct expressions for a volume of revolution. Common errors of course included the omission of dx, using the given domain as the upper and lower bounds of integration, forgetting to square their function and/or the omission of π . There were still many who were unable to use their calculator successfully to find the required volume.

Let $f(x) = 4x - \mathrm{e}^{x-2} - 3$, for $0 \le x \le 5$.

a. Find the x-intercepts of the graph of f . [3]

b. On the grid below, sketch the graph of f . [3]



c. Write down the gradient of the graph of f at $x = 3$. [1]

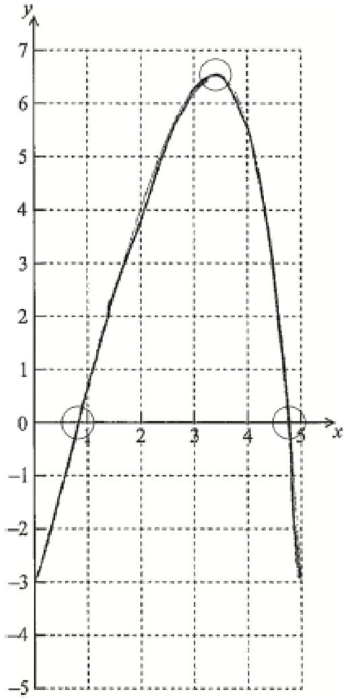
Markscheme

a. intercepts when $f(x) = 0$ ***M1***

(0.827, 0) (4.78, 0) (accept $x = 0.827$, $x = 4.78$) ***A1A1*** ***N3***

[3 marks]

b.



A1A1A1 ***N3***

Note: Award ***A1*** for maximum point in circle, ***A1*** for *x*-intercepts in circles, ***A1*** for correct shape (*y* approximately greater than -3.14).

[3 marks]

c. gradient is 1.28 ***A1*** ***N1***

[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = ax^3 + bx^2 + c$, where a , b and c are real numbers. The graph of *f* passes through the point (2, 9) .

a. Show that $8a + 4b + c = 9$. [2]

b. The graph of *f* has a local minimum at $(1, 4)$. [7]

Find two other equations in a , b and c , giving your answers in a similar form to part (a).

c. Find the value of a , of b and of c . [4]

Markscheme

a. attempt to substitute coordinates in f ***(M1)***

e.g. $f(2) = 9$

correct substitution ***A1***

e.g. $a \times 2^3 + b \times 2^2 + c = 9$

$8a + 4b + c = 9$ ***AG*** ***N0***

[2 marks]

b. recognizing that $(1, 4)$ is on the graph of f ***(M1)***

e.g. $f(1) = 4$

correct equation ***A1***

e.g. $a + b + c = 4$

recognizing that $f' = 0$ at minimum (seen anywhere) ***(M1)***

e.g. $f'(1) = 0$

$f'(x) = 3ax^2 + 2bx$ (seen anywhere) ***A1A1***

correct substitution into derivative ***(A1)***

e.g. $3a \times 1^2 + 2b \times 1 = 0$

correct simplified equation ***A1***

e.g. $3a + 2b = 0$

[7 marks]

c. valid method for solving system of equations ***(M1)***

e.g. inverse of a matrix, substitution

$a = 2$, $b = -3$, $c = 5$ ***A1A1A1*** ***N4***

[4 marks]

Examiners report

a. Part (a) was generally well done, with a few candidates failing to show a detailed substitution. Some substituted 2 in place of x, but didn't make it clear that they had substituted in y as well.

b. A great majority could find the two equations in part (b). However there were a significant number of candidates who failed to identify that the gradient of the tangent is zero at a minimum point, thus getting the incorrect equation $3a + 2b = 4$.

c. A considerable number of candidates only had 2 equations, so that they either had a hard time trying to come up with a third equation (incorrectly combining some of the information given in the question) to solve part (c) or they completely failed to solve it.

Despite obtaining three correct equations many used long elimination methods that caused algebraic errors. Pages of calculations leading nowhere were seen.

Those who used matrix methods were almost completely successful.

Consider the graph of $f(x) = \frac{e^x}{5x-10} + 3$, for $x \neq 2$.

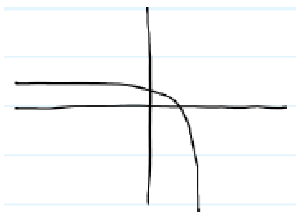
a. Find the y-intercept. [2]

b. Find the equation of the vertical asymptote. [2]

c. Find the minimum value of $f(x)$ for $x > 2$. [2]

Markscheme

a. valid approach ***(M1)***

eg $f(0)$, 

y-intercept is 2.9 ***A1*** ***N2***

[2 marks]

b. valid approach involving equation or inequality ***(M1)***

eg $5x - 10 = 0,\ 2,\ x \neq 2$

$x = 2$ (must be an equation) ***A1*** ***N2***

[2 marks]

c. 7.01710

$\text{min value} = 7.02$ ***A2*** ***N2***

Note: If candidate gives the minimum point as their final answer, award ***A1*** for $(3,\ 7.02)$.

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the function $f(x) = x^2 - 4x + 1$.

a. Sketch the graph of f , for $-1 \leq x \leq 5$. [4]

b. This function can also be written as $f(x) = (x - p)^2 - 3$. [1]

Write down the value of p .

c. The graph of g is obtained by reflecting the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. [4]

Show that $g(x) = -x^2 + 4x + 5$.

d. The graph of g is obtained by reflecting the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. [3]

The graphs of f and g intersect at two points.

Write down the x-coordinates of these two points.

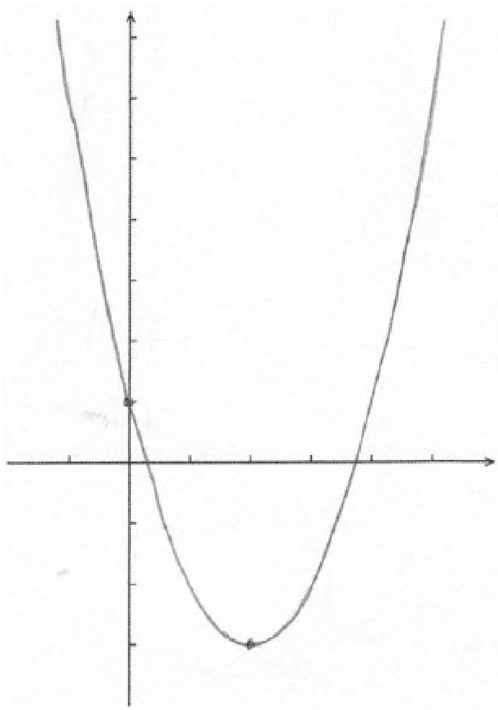
e. The graph of g is obtained by reflecting the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. [3]

Let R be the region enclosed by the graphs of f and g .

Find the area of R .

Markscheme

a.



A1A1A1A1 *N4*

Note: The shape **must** be an approximately correct upwards parabola.

Only if the shape is approximately correct, award the following:

A1 for vertex $x \approx 2$, ***A1*** for x-intercepts between 0 and 1, and 3 and 4, ***A1*** for correct y-intercept $(0, 1)$, ***A1*** for correct domain $[-1, 5]$.

Scale not required on the axes, but approximate positions need to be clear.

[4 marks]

b. $p = 2$ ***A1*** ***N1***

[1 mark]

c. correct vertical reflection, correct vertical translation ***(A1)(A1)***

e.g. $-f(x)$, $-((x-2)^2 - 3)$, $-y$, $-f(x) + 6$, $y + 6$

transformations in correct order ***(A1)***

e.g. $-(x^2 - 4x + 1) + 6$, $-((x-2)^2 - 3) + 6$

simplification which clearly leads to given answer ***A1*** e.g. $-x^2+4x-1+6$, $-(x^2-4x+4-3)+6$

$g(x)=-x^2+4x+5$ ***AG*** ***N0***

Note: If working shown, award ***A1A1A0A0*** if transformations correct, but done in reverse order, e.g. $-(x^2-4x+1+6)$.

[4 marks]

d. valid approach ***(M1)***

e.g. sketch, $f=g$

$-0.449489\ldots$, $4.449489\ldots$

$(2\pm\sqrt{6})$ (exact), -0.449 $[-0.450,\ -0.449]$; 4.45 $[4.44, 4.45]$ ***A1A1*** ***N3***

[3 marks]

e. attempt to substitute limits or functions into area formula (accept absence of $\mathrm{d}x$) ***(M1)***

e.g. $\int_a^b((-x^2+4x+5)-(x^2-4x+1))\mathrm{d}x$, $\int_{4.45}^{-0.449}(f-g)$, $\int(-2x^2+8x+4)\mathrm{d}x$

approach involving subtraction of integrals/areas (accept absence of $\mathrm{d}x$) ***(M1)***

e.g. $\int_a^b(-x^2+4x+5)-\int_a^b(x^2-4x+1)$, $\int(f-g)\mathrm{d}x$

area $=39.19183\ldots$

area $=39.2$ $[39.1, 39.2]$ ***A1*** ***N3***

[3 marks]

Examiners report

a. A good number of students provided a clear sketch of the quadratic function within the given domain. Some lost marks as they did not clearly indicate the approximate positions of the most important points of the parabola either by labelling or providing a suitable scale.

b. There were few difficulties in part (b).

c. In part (c), candidates often used an insufficient number of steps to show the required result or had difficulty setting out their work logically.

d. Part (d) was generally done well though many candidates gave at least one answer to fewer than three significant figures, potentially resulting in more lost marks.

e. In part (e), many candidates were unable to connect the points of intersection found in part (d) with the limits of integration. Mistakes were also made here either using a GDC incorrectly or not subtracting the correct functions. Other candidates tried to divide the region into four areas and made obvious errors in the process. Very few candidates subtracted $f(x)$ from $g(x)$ to get a simple function before integrating and there were numerous, fruitless analytical attempts to find the required integral.

Let $f(x)=2x^2-8x-9$.

a(i) and (ii). (i) Write down the coordinates of the vertex. [4]

(ii) Hence or otherwise, express the function in the form $f(x)=2(x-h)^2+k$.

b. Solve the equation $f(x) = 0$. [3]

Markscheme

a(i) and (ii). (i) $(2, -17)$ or $x = 2$, $y = -17$ ***A1A1 N2***

(ii) evidence of valid approach ***(M1)***

e.g. graph, completing the square, equating coefficients

$f(x) = 2(x-2)^2 - 17$ ***A1 N2***

[4 marks]

b. evidence of valid approach ***(M1)***

e.g. graph, quadratic formula

$-0.9154759\ldots, 4.915475\ldots$

$x = -0.915$, 4.92 ***A1A1 N3***

[3 marks]

Examiners report

a(i) and (ii). This question was well done by the majority of candidates.

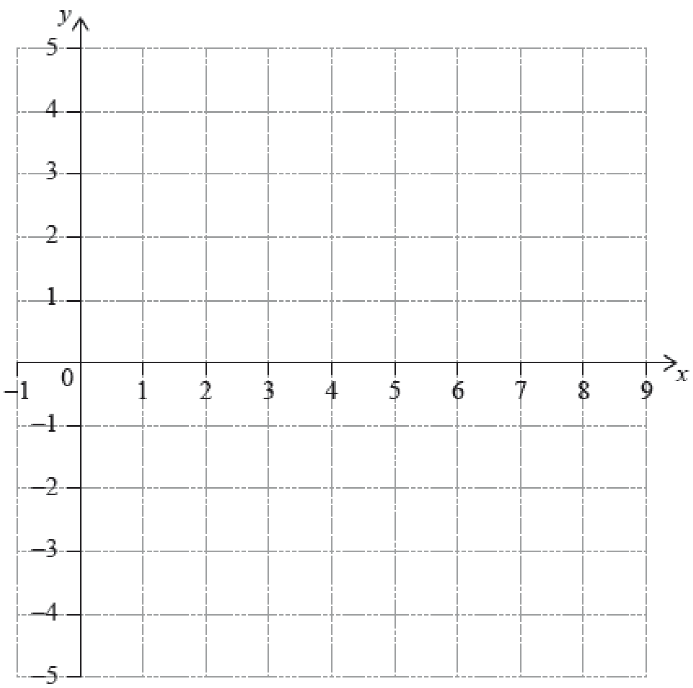
b. This question was well done by the majority of candidates. There were still many however who opted for an analytical approach in part (b), which often led to errors in sign and accuracy. Some candidates used the trace feature on their GDC to find the vertex which often resulted in accuracy errors.

Let $f(x) = \frac{6x^2-4}{e^x}$, for $0 \leqslant x \leqslant 7$.

a. Find the x-intercept of the graph of f. [2]

b. The graph of f has a maximum at the point A. Write down the coordinates of A. [2]

c. On the following grid, sketch the graph of f. [3]



Markscheme

a. valid approach *(M1)*

eg $f(x) = 0, \ \pm 0.816$

0.816496

$x = \sqrt{\frac{2}{3}}$ (exact), 0.816 ***A1*** ***N2***

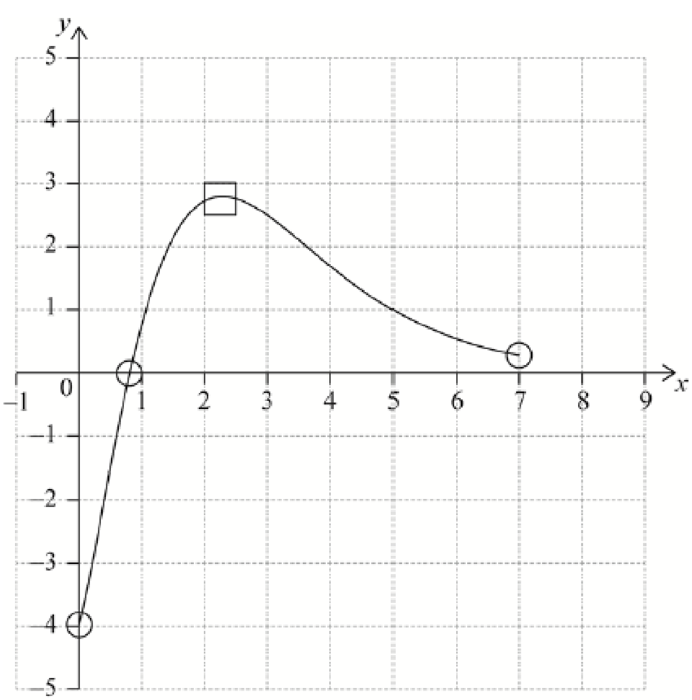
[2 marks]

b. $(2.29099, \ 2.78124)$

$\mathrm{A}(2.29, \ 2.78)$ ***A1A1*** ***N2***

[2 marks]

c.



A1A1A1 ***N3***

Notes: Award ***A1*** for correct domain and endpoints at $x = 0$ and $x = 7$ in circles,

A1 for maximum in square,

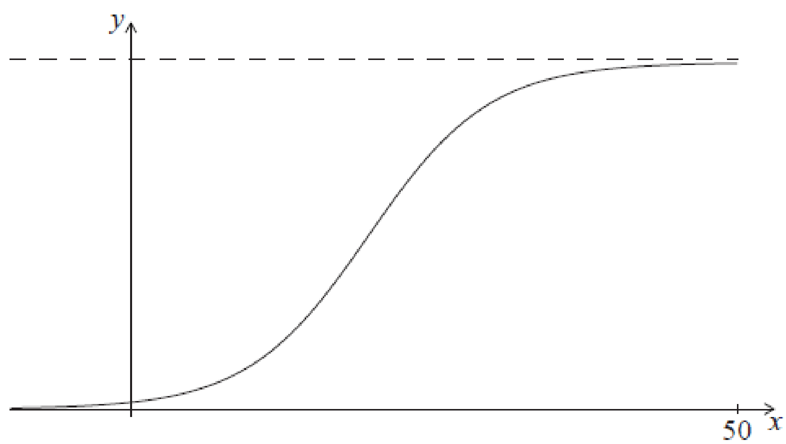
A1 for approximately correct shape that passes through **their** x-intercept in circle and has changed from concave down to concave up between 2.29 and 7.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = \frac{100}{(1+50e^{-0.2x})}$. Part of the graph of f is shown below.



a. Write down $f(0)$. [1]

b. Solve $f(x) = 95$. [2]

c. Find the range of f . [3]

d. Show that $f'(x) = \frac{1000e^{-0.2x}}{(1+50e^{-0.2x})^2}$. [5]

e. Find the maximum rate of change of f . [4]

Markscheme

a. $f(0) = \frac{100}{51}$ (exact), 1.96 ***A1 N1***

[1 mark]

b. setting up equation ***(M1)***

eg $95 = \frac{100}{1+50e^{-0.2x}}$, sketch of graph with horizontal line at $y = 95$

$x = 34.3$ ***A1 N2***

[2 marks]

c. upper bound of y is 100 ***(A1)***

lower bound of y is 0 ***(A1)***

range is $0 < y < 100$ ***A1*** ***N3***

[3 marks]

d. **METHOD 1**

setting function ready to apply the chain rule ***(M1)***

eg $100(1+50\mathrm{e}^{-0.2x})^{-1}$

evidence of correct differentiation (must be substituted into chain rule) ***(A1)(A1)***

eg $u' = -100(1+50\mathrm{e}^{-0.2x})^{-2}$, $v' = (50\mathrm{e}^{-0.2x})(-0.2)$

correct chain rule derivative ***A1***

eg $f'(x) = -100(1+50\mathrm{e}^{-0.2x})^{-2}(50\mathrm{e}^{-0.2x})(-0.2)$

correct working clearly leading to the required answer ***A1***

eg $f'(x) = 1000\mathrm{e}^{-0.2x}(1+50\mathrm{e}^{-0.2x})^{-2}$

$f'(x) = \frac{1000\mathrm{e}^{-0.2x}}{(1+50\mathrm{e}^{-0.2x})^2}$ ***AG*** ***N0***

METHOD 2

attempt to apply the quotient rule (accept reversed numerator terms) ***(M1)***

eg $\frac{vu'-uv'}{v^2}$, $\frac{uv'-vu'}{v^2}$

evidence of correct differentiation inside the quotient rule ***(A1)(A1)***

eg $f'(x) = \frac{(1+50\mathrm{e}^{-0.2x})(0)-100(50\mathrm{e}^{-0.2x}\times-0.2)}{(1+50\mathrm{e}^{-0.2x})^2}$, $\frac{100(-10)\mathrm{e}^{-0.2x}-0}{(1+50\mathrm{e}^{-0.2x})^2}$

any correct expression for derivative (0 may not be explicitly seen) ***(A1)***

eg $\frac{-100(50\mathrm{e}^{-0.2x}\times-0.2)}{(1+50\mathrm{e}^{-0.2x})^2}$

correct working clearly leading to the required answer ***A1***

eg $f'(x) = \frac{0-100(-10)\mathrm{e}^{-0.2x}}{(1+50\mathrm{e}^{-0.2x})^2}$, $\frac{-100(-10)\mathrm{e}^{-0.2x}}{(1+50\mathrm{e}^{-0.2x})^2}$

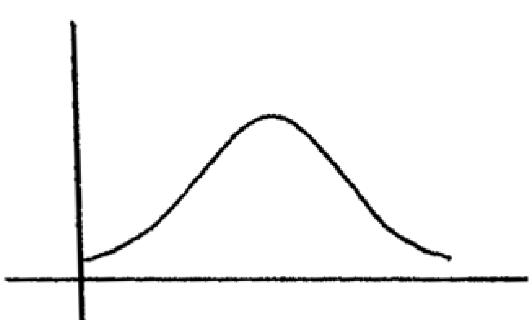
$f'(x) = \frac{1000\mathrm{e}^{-0.2x}}{(1+50\mathrm{e}^{-0.2x})^2}$ ***AG*** ***N0***

[5 marks]

e. **METHOD 1**

sketch of $f'(x)$ ***(A1)***

eg



recognizing maximum on $f'(x)$ ***(M1)***

eg dot on max of sketch

finding maximum on graph of $f'(x)$ ***A1*** *eg* $(19.6, 5)$, $x = 19.560\ldots$

maximum rate of increase is 5 ***A1 N2***

METHOD 2

recognizing $f''(x) = 0$ ***(M1)***

finding any correct expression for $f''(x) = 0$ ***(A1)***

eg $\frac{(1+50\mathrm{e}^{-0.2x})^2(-200\mathrm{e}^{-0.2x})-(1000\mathrm{e}^{-0.2x})(2(1+50\mathrm{e}^{-0.2x})(-10\mathrm{e}^{-0.2x}))}{(1+50\mathrm{e}^{-0.2x})^4}$

finding $x = 19.560\ldots$ ***A1***

maximum rate of increase is 5 ***A1 N2***

[4 marks]

Examiners report

a. Candidates had little difficulty with parts (a), (b) and (c).

b. Candidates had little difficulty with parts (a), (b) and (c). Successful analytical approaches were often used in part (b) but again, this was not the most efficient or expected method.

c. Candidates had little difficulty with parts (a), (b) and (c). In part (c), candidates gained marks by correctly identifying upper and lower bounds but often did not express them properly using an appropriate notation.

d. In part (d), the majority of candidates opted to use the quotient rule and did so with some degree of competency, but failed to recognize the command term "show that" and consequently did not show enough to gain full marks. Approaches involving the chain rule were also successful but with the same point regarding sufficiency of work.

e. Part (e) was poorly done as most were unable to interpret what was required. There were a few responses involving the use of the "trace" feature of the GDC which often led to inaccurate answers and a number of candidates incorrectly reported $x = 19.6$ as their final answer. Some found the maximum value of f rather than f'.

Let $f(x) = \frac{8x-5}{cx+6}$ for $x \neq -\frac{6}{c},\ c \neq 0.$

a. The line *x* = 3 is a vertical asymptote to the graph of *f*. Find the value of *c*. [2]

b. Write down the equation of the horizontal asymptote to the graph of *f*. [2]

c. The line *y* = *k*, where $k \in \mathbb{R}$ intersects the graph of $|f(x)|$ at exactly one point. Find the possible values of *k*. [3]

Markscheme

a. valid approach ***(M1)***

eg $cx + 6 = 0,\ -\frac{6}{c} = 3$

$c = -2$ ***A1 N2***

[2 marks]

b. valid approach ***(M1)***

eg $\lim\limits_{x\to\infty} f(x),\ y = \frac{8}{c}$

$y = -4$ (must be an equation) ***A1 N2***

[2 marks]

c. valid approach to analyze modulus function ***(M1)***

eg sketch, horizontal asymptote at $y = 4$, $y = 0$

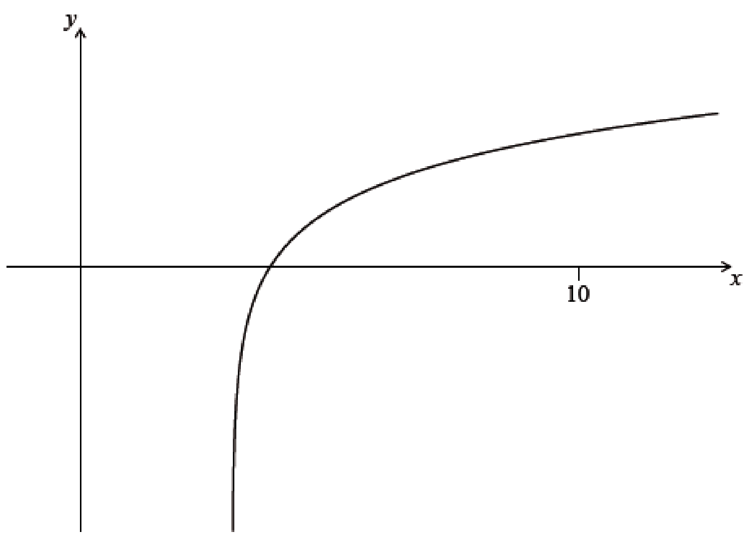
$k = 4$, $k = 0$ ***A2 N3***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = 2\ln(x-3)$, for $x > 3$. The following diagram shows part of the graph of f.



a. Find the equation of the vertical asymptote to the graph of f. [2]

b. Find the x-intercept of the graph of f. [2]

c. The region enclosed by the graph of f, the x-axis and the line $x = 10$ is rotated $360°$ about the x-axis. Find the volume of the solid formed. [3]

Markscheme

a. valid approach ***(M1)***

eg horizontal translation 3 units to the right

$x = 3$ (must be an equation) ***A1 N2***

[2 marks]

b. valid approach ***(M1)***

eg $f(x) = 0,\ e^0 = x - 3$

$4,\ x = 4,\ (4,\ 0)$ ***A1*** ***N2***

[2 marks]

c. attempt to substitute either **their correct** limits or the function into formula involving f^2 ***(M1)***

eg $\int_4^{10} f^2,\ \pi \int (2\ln(x-3))^2 \mathrm{d}x$

141.537

volume = 142 ***A2*** ***N3***

[3 marks]

Total [7 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = \cos(x^2)$ and $g(x) = \mathrm{e}^x$, for $-1.5 \le x \le 0.5$.

Find the area of the region enclosed by the graphs of *f* and *g* .

Markscheme

evidence of finding intersection points ***(M1)***

e.g. $f(x) = g(x)$, $\cos x^2 = \mathrm{e}^x$, sketch showing intersection

$x = -1.11$, $x = 0$ (may be seen as limits in the integral) ***A1A1***

evidence of approach involving integration and subtraction (in any order) ***(M1)***

e.g. $\int_{-1.11}^{0} \cos x^2 - \mathrm{e}^x$, $\int (\cos x^2 - \mathrm{e}^x)\mathrm{d}x$, $\int g - f$

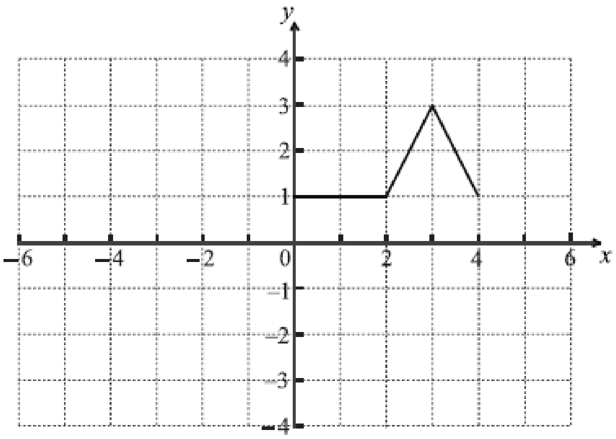
area $= 0.282$ ***A2*** ***N3***

[6 marks]

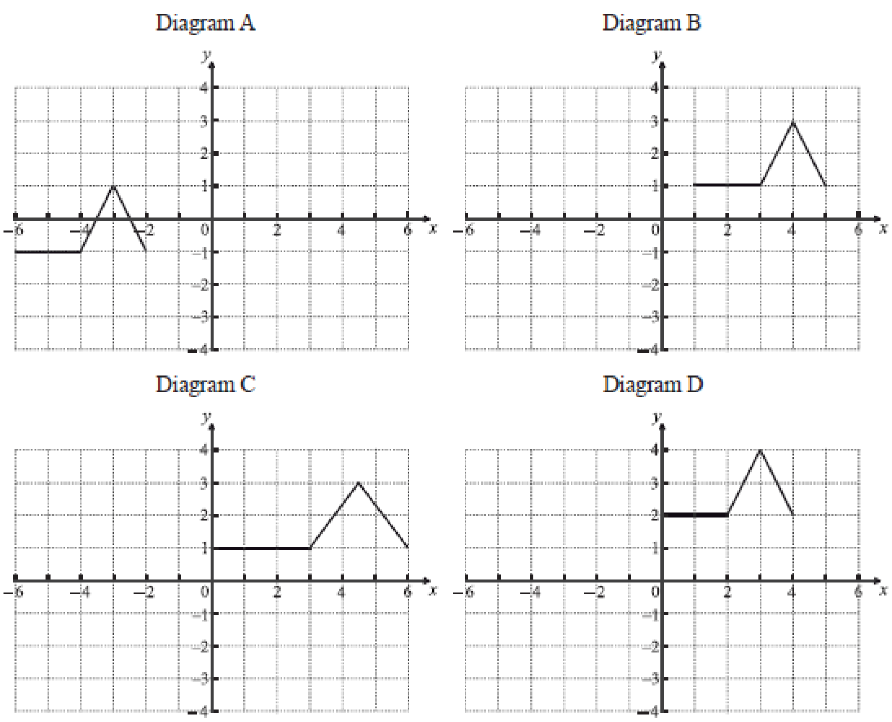
Examiners report

This question was poorly done by a great many candidates. Most seemed not to understand what was meant by the phrase "region enclosed by" as several candidates assumed that the limits of the integral were those given in the domain. Few realized what area was required, or that intersection points were needed. Candidates who used their GDCs to first draw a suitable sketch could normally recognize the required region and could find the intersection points correctly. However, it was disappointing to see the number of candidates who could not then use their GDC to find the required area or who attempted unsuccessful analytical approaches.

Consider the graph of f shown below.



The following four diagrams show **images** of f under different transformations.



a. On the **same** grid sketch the graph of $y = f(-x)$. [2]

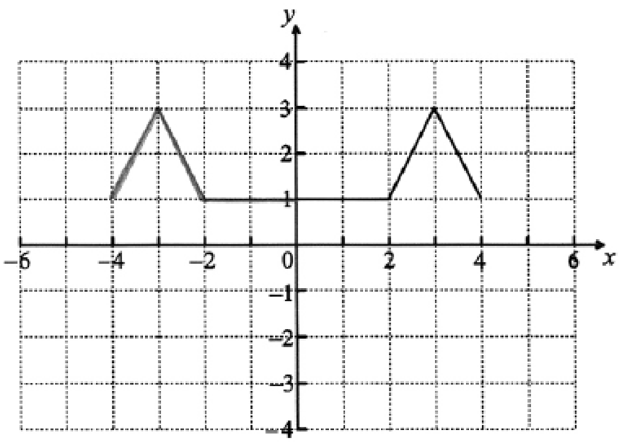
b. Complete the following table. [2]

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	
Maps f to $f(x)+1$	

c. Give a full geometric description of the transformation that gives the image in Diagram A. [2]

Markscheme

a.



A2 N2

[2 marks]

b.

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	**C**
Maps f to $f(x)+1$	**D**

A1A1 N2

[2 marks]

c. translation (accept move/shift/slide etc.) with vector $\begin{pmatrix} -6 \\ -2 \end{pmatrix}$ *A1A1 N2*

[2 marks]

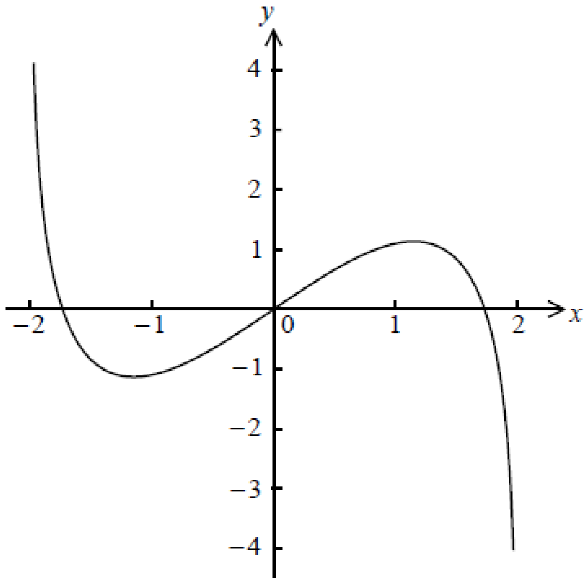
Examiners report

a. This question was reasonably solved by many students, though a good number confused $f(-x)$ with $-f(x)$ in part (a), thus reflecting the original diagram in the *x*-axis. Candidates need more practice in correctly and fully describing transformations.

b. Candidates need more practice in correctly and fully describing transformations. There was often confusion between the description of the transformation and the equation that represents it. A fairly low percentage of the candidates used the term "translation".

c. Candidates need more practice in correctly and fully describing transformations. There was often confusion between the description of the transformation and the equation that represents it. A fairly low percentage of the candidates used the term "translation".

Consider $f(x) = x\ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is given below.



a(i) and (ii). Let P and Q be points on the curve of f where the tangent to the graph of f is parallel to the x-axis. [5]

(i) Find the x-coordinate of P and of Q.

(ii) Consider $f(x) = k$. Write down all values of k for which there are exactly two solutions.

b. Let $g(x) = x^3 \ln(4 - x^2)$, for $-2 < x < 2$. [4]

Show that $g'(x) = \frac{-2x^4}{4-x^2} + 3x^2 \ln(4 - x^2)$.

c. Let $g(x) = x^3 \ln(4 - x^2)$, for $-2 < x < 2$. [2]

Sketch the graph of g' .

d. Let $g(x) = x^3 \ln(4 - x^2)$, for $-2 < x < 2$. [3]

Consider $g'(x) = w$. Write down all values of w for which there are exactly two solutions.

Markscheme

a(i) and (ii). (i) -1.15, 1.15 ***A1A1*** ***N2***

(ii) recognizing that it occurs at P and Q ***(M1)***

e.g. $x = -1.15$, $x = 1.15$

$k = -1.13$, $k = 1.13$ ***A1A1*** ***N3***

[5 marks]

b. evidence of choosing the product rule ***(M1)***

e.g. $uv' + vu'$

derivative of x^3 is $3x^2$ ***(A1)***

derivative of $\ln(4 - x^2)$ is $\frac{-2x}{4-x^2}$ ***(A1)***

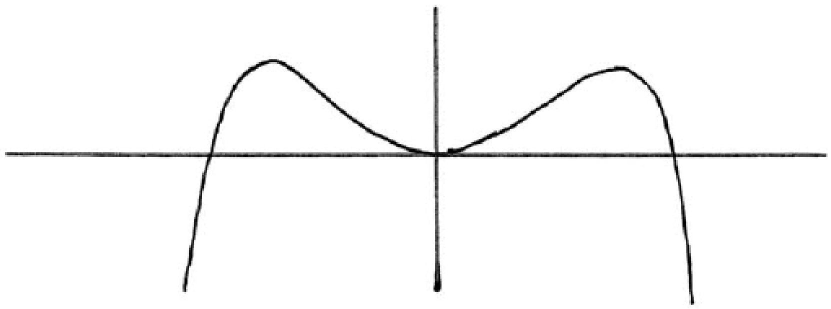
correct substitution ***A1***

e.g. $x^3 \times \frac{-2x}{4-x^2} + \ln(4-x^2) \times 3x^2$

$g'(x) = \frac{-2x^4}{4-x^2} + 3x^2 \ln(4-x^2)$ ***AG N0***

[4 marks]

c.



A1A1 N2

[2 marks]

d. $w = 2.69$, $w < 0$ ***A1A2 N2***

[3 marks]

Examiners report

a(i) and (ii). Many candidates correctly found the *x*-coordinates of P and Q in (a)(i) with their GDC. In (a)(ii) some candidates incorrectly interpreted the words "exactly two solutions" as an indication that the discriminant of a quadratic was required. Many failed to realise that the values of *k* they were looking for in this question were the *y*-coordinates of the points found in (a)(i).

b. Many candidates were unclear in their application of the product formula in the verifying the given derivative of *g*. Showing that the derivative was the given expression often received full marks though it was not easy to tell in some cases if that demonstration came through understanding of the product and chain rules or from reasoning backwards from the given result.

c. Some candidates drew their graphs of the derivative in (c) on their examination papers despite clear instructions to do their work on separate sheets. Most who tried to plot the graph in (c) did so successfully.

d. Correct solutions to 10(d) were not often seen.

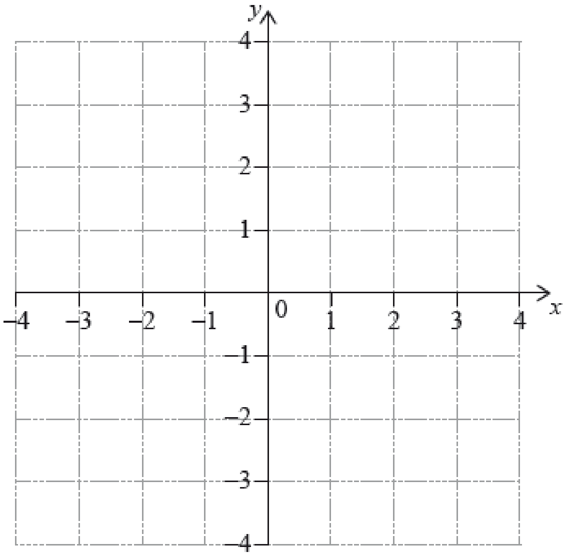
Let $f(x) = 0.225x^3 - 2.7x$, for $-3 \leqslant x \leqslant 3$. There is a local minimum point at A.

On the following grid,

a. Find the coordinates of A. [2]

b. (i) sketch the graph of f, clearly indicating the point A; [5]

(ii) sketch the tangent to the graph of f at A.



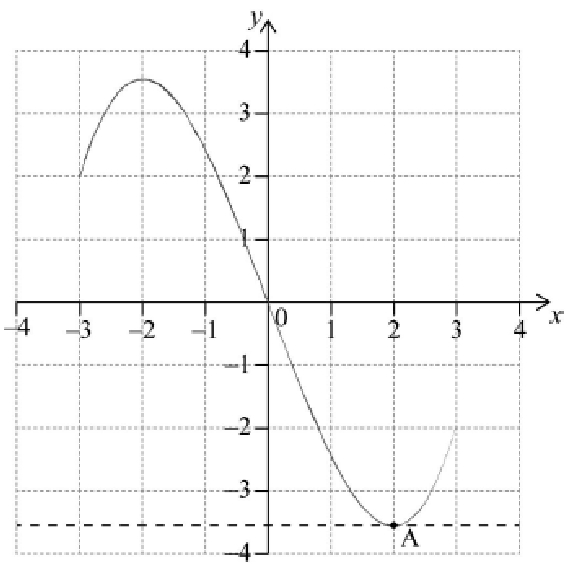
Markscheme

a. $\mathrm{A}\ (2,\ -3.6)$ ***A1A1*** ***N2***

[2 marks]

b.

(i) (ii)



A1

A1A1A1 ***N4***

A1 ***N1***

Notes: (i) Award ***A1*** for correct cubic shape with correct curvature.

Only if this ***A1*** is awarded, award the following:

A1 for passing through **their** point A and the origin,

A1 for endpoints,

A1 for maximum.

(ii) Award ***A1*** for horizontal line through **their** A.

[5 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = x\mathrm{e}^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

a. Find the value of p and of q. [3]

b. Hence, find the area of the region enclosed by the graphs of f and g. [3]

Markscheme

a. valid attempt to find the intersection ***(M1)***

eg $f = g$, sketch, one correct answer

$p = 0.357402,\ q = 2.15329$

$p = 0.357,\ q = 2.15$ ***A1A1 N3***

[3 marks]

b. attempt to set up an integral involving subtraction (in any order) ***(M1)***

eg $\int_p^q [f(x) - g(x)]\,\mathrm{d}x,\ \int_p^q f(x)\mathrm{d}x - \int_p^q g(x)\mathrm{d}x$

0.537667

$\text{area} = 0.538$ ***A2 N3***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = \ln x - 5x$, for $x > 0$.

a. Find $f'(x)$. [2]

b. Find $f''(x)$. [1]

c. Solve $f'(x) = f''(x)$. [2]

Markscheme

a. $f'(x) = \frac{1}{x} - 5$ ***A1A1 N2***

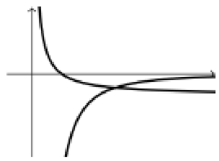
[2 marks]

b. $f''(x) = -x^{-2}$ ***A1 N1***

[1 mark]

c. **METHOD 1 (using GDC)**

valid approach ***(M1)***

eg  $, -0.358$

0.558257

$x = 0.558$ ***A1 N2***

Note: Do not award ***A1*** if additional answers given.

METHOD 2 (analytical)

attempt to solve their equation $f'(x) = f''(x)$ (do not accept $\frac{1}{x} - 5 = -\frac{1}{x^2}$) ***(M1)***

eg $5x^2 - x - 1 = 0,\ \frac{1 \pm \sqrt{21}}{10},\ \frac{1}{x} = \frac{-1 \pm \sqrt{21}}{2},\ -0.358$

0.558257

$x = 0.558$ ***A1 N2***

Note: Do not award ***A1*** if additional answers given.

[2 marks]

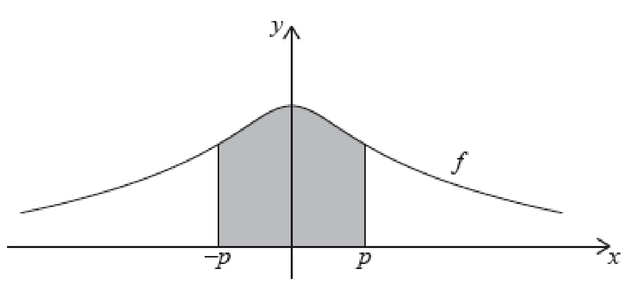
Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of f passes through the point $(p,\ 4)$, where $p > 0$.

a. Find the value of p. [2]

b. The following diagram shows part of the graph of f. [3]



The region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$ is rotated 360° about the x-axis. Find the volume of the solid formed.

Markscheme

a. valid approach *(M1)*

eg $f(p) = 4$, intersection with $y = 4$, ± 2.32

2.32143

$p = \sqrt{e^2 - 2}$ (exact), 2.32 ***A1*** ***N2***

[2 marks]

b. attempt to substitute **either their** limits **or** the function into volume formula (must involve f^2, accept reversed limits and absence of π and/or dx, but do not accept any other errors) *(M1)*

eg $\int_{-2.32}^{2.32} f^2$, $\pi \int \left(6 - \ln(x^2 + 2)\right)^2 dx$, 105.675

331.989

$\text{volume} = 332$ ***A2*** ***N3***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Let f and g be functions such that $g(x) = 2f(x + 1) + 5$.

. (a) The graph of f is mapped to the graph of g under the following transformations: [6]

vertical stretch by a factor of k , followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k ;

(ii) p ;

(iii) q .

(b) Let $h(x) = -g(3x)$. The point A(6, 5) on the graph of g is mapped to the point A' on the graph of h . Find A' .

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vertical stretch by a factor of k , followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k ;

(ii) p ;

(iii) q .

b. Let $h(x) = -g(3x)$. The point A(6, 5) on the graph of g is mapped to the point A' on the graph of h . Find A' . [3]

Markscheme

. (a) (i) $k = 2$ ***A1*** ***N1***

(ii) $p = -1$ ***A1*** ***N1***

(iii) $q = 5$ ***A1*** ***N1***

[3 marks]

(b) recognizing one transformation ***(M1)***

eg horizontal stretch by $\frac{1}{3}$, reflection in x-axis

A$'$ is (2, -5) ***A1A1*** ***N3***

[3 marks]

Total [6 marks]

a. (i) $k = 2$ ***A1*** ***N1***

(ii) $p = -1$ ***A1*** ***N1***

(iii) $q = 5$ ***A1*** ***N1***

[3 marks]

b. recognizing one transformation ***(M1)***

eg horizontal stretch by $\frac{1}{3}$, reflection in x-axis

A$'$ is (2, -5) ***A1A1*** ***N3***

[3 marks]

Total [6 marks]

Examiners report

. Part (a) was frequently done well but a lack of understanding of the notation $f(x + 1)$ often led to an incorrect value for p. In part (b), candidates did not recognize the simplicity of the problem. Most seemed to be unable to correctly recognize the two transformations implied in the question and were thus unable to attempt a geometric solution. Flawed algebraic approaches to part (b) were common and many could not interpret the notation $g(3x)$ as multiplying the x-value by $\frac{1}{3}$.

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Let $f(x) = \frac{3x}{x-q}$, where $x \neq q$.

a. Write down the equations of the vertical and horizontal asymptotes of the graph of f. [2]

b. The vertical and horizontal asymptotes to the graph of f intersect at the point $\mathrm{Q}(1, 3)$. [2]

Find the value of q.

c. The vertical and horizontal asymptotes to the graph of f intersect at the point $\mathrm{Q}(1, 3)$. [4]

The point $\mathrm{P}(x,\ y)$ lies on the graph of f. Show that $\mathrm{PQ} = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$.

d. The vertical and horizontal asymptotes to the graph of f intersect at the point $\mathrm{Q}(1, 3)$. [6]

Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$.

Markscheme

a. $x = q,\ y = 3$ (must be equations) ***A1A1*** ***N2***

[2 marks]

b. recognizing connection between point of intersection and asymptote ***(R1)***

eg $x = 1$

$q = 1$ ***A1*** ***N2***

[2 marks]

c. correct substitution into distance formula ***A1***

eg $\sqrt{(x-1)^2 + (y-3)^2}$

attempt to substitute $y = \frac{3x}{x-1}$ ***(M1)***

eg $\sqrt{(x-1)^2 + \left(\frac{3x}{x-1} - 3\right)^2}$

correct simplification of $\left(\frac{3x}{x-1} - 3\right)$ ***(A1)***

eg $\frac{3x-3x(x-1)}{x-1}$

correct expression clearly leading to the required answer ***A1***

eg $\frac{3x-3x+3}{x-1}$, $\sqrt{(x-1)^2 + \left(\frac{3x-3x+3}{x-1}\right)^2}$

$\mathrm{PQ} = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$ ***AG*** ***N0***

[4 marks]

d. recognizing that closest is when PQ is a minimum ***(R1)***

eg sketch of PQ, $(\mathrm{PQ})'(x) = 0$

$x = -0.73205$ $x = 2.73205$ (seen anywhere) ***A1A1***

attempt to find *y*-coordinates ***(M1)***

eg $f(-0.73205)$

$(-0.73205, 1.267949), (2.73205, 4.73205)$

$(-0.732, 1.27), (2.73, 4.73)$ ***A1A1*** ***N4***

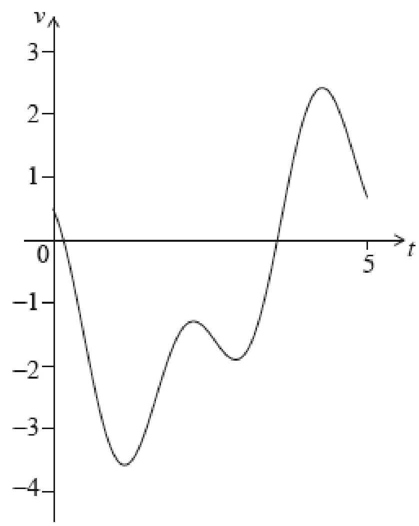
[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

A particle P moves along a straight line so that its velocity, $v\,\mathrm{ms}^{-1}$, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \leqslant t \leqslant 5$. The initial displacement of P from a fixed point O is 4 metres.

The following sketch shows the graph of v.



a. Find the displacement of P from O after 5 seconds. [5]

b. Find when P is first at rest. [2]

c. Write down the number of times P changes direction. [2]

d. Find the acceleration of P after 3 seconds. [2]

e. Find the maximum speed of P. [3]

Markscheme

a. **METHOD 1**

recognizing $s = \int v$ ***(M1)***

recognizing displacement of P in first 5 seconds (seen anywhere) ***A1***

(accept missing $\mathrm{d}t$)

eg $\int_0^5 v\mathrm{d}t, \ -3.71591$

valid approach to find total displacement ***(M1)***

eg $4 + (-3.7159), \ s = 4 + \int_0^5 v$

0.284086

0.284 (m) ***A2*** ***N3***

METHOD 2

recognizing $s = \int v$ ***(M1)***

correct integration ***A1***

eg $\frac{1}{3}\sin 3t + 2\cos t - \frac{t}{2} + c$ (do not penalize missing "c")

attempt to find c ***(M1)***

eg $4 = \frac{1}{3}\sin(0) + 2\cos(0) - -\frac{0}{2} + c,\ 4 = \frac{1}{3}\sin 3t + 2\cos t - \frac{t}{2} + c,\ 2 + c = 4$

attempt to substitute $t = 5$ into their expression with c ***(M1)***

eg $s(5),\ \frac{1}{3}\sin(15) + 2\cos(5)5 - -\frac{5}{2} + 2$

0.284086

0.284 (m) ***A1*** ***N3***

[5 marks]

b. recognizing that at rest, $v = 0$ ***(M1)***

$t = 0.179900$

$t = 0.180\ (\text{secs})$ ***A1*** ***N2***

[2 marks]

c. recognizing when change of direction occurs ***(M1)***

eg v crosses t axis

2 (times) ***A1*** ***N2***

[2 marks]

d. acceleration is v' (seen anywhere) ***(M1)***

eg $v'(3)$

0.743631

$0.744\ (\text{ms}^{-2})$ ***A1*** ***N2***

[2 marks]

e. valid approach involving max or min of v ***(M1)***

eg $v\prime = 0,\ a = 0$, graph

one correct co-ordinate for min ***(A1)***

eg $1.14102,\ -3.27876$

$3.28\ (\text{ms}^{-1})$ ***A1*** ***N2***

[3 marks]

Examiners report

a. This question was not well done throughout. Analytical approaches were almost always unsuccessful as a result of poor integration and differentiation skills and many of the errors were a result of having the GDC in degree mode. In (a), most candidates recognized the need to integrate v to find the displacement, although a significant number differentiated v. Of those that integrated, many assumed incorrectly that the initial displacement was the value of the constant of integration. Some candidates integrated $|v|$ and obtained no marks for an invalid approach. In the case where a correct definite integral was given, it was disappointing to see many candidates try to evaluate it analytically rather than using their GDC.

b. This question was not well done throughout. Analytical approaches were almost always unsuccessful as a result of poor integration and differentiation skills and many of the errors were a result of having the GDC in degree mode. In part (b), many candidates did not read the question carefully and gave the two occasions, in the given domain, where the particle was at rest.

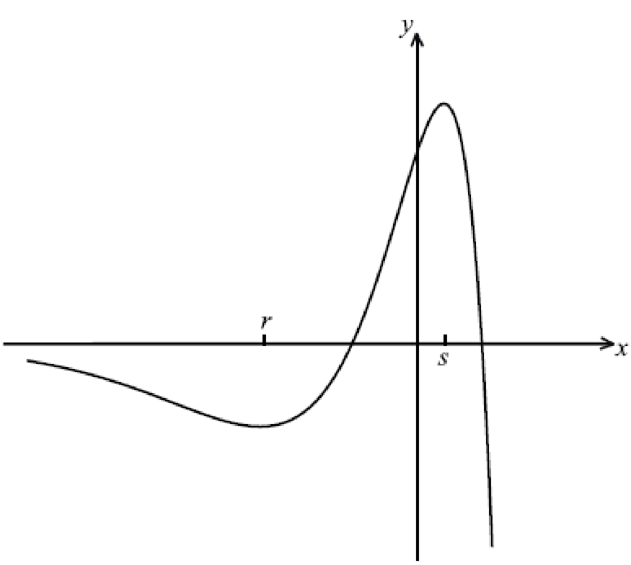
c. This question was not well done throughout. Analytical approaches were almost always unsuccessful as a result of poor integration and differentiation skills and many of the errors were a result of having the GDC in degree mode. In part (c), many candidates did not appreciate that velocity is a vector and that the particle would change direction when its velocity changes sign. Consequently, many candidates gave the incorrect answer of four changes in directions, rather than the correct two direction changes.

d. This question was not well done throughout. Analytical approaches were almost always unsuccessful as a result of poor integration and differentiation skills and many of the errors were a result of having the GDC in degree mode. Part (d), was done very poorly, with candidates struggling to differentiate sine and cosine correctly and to evaluate their derivative. As with question 3, many candidates worked with the incorrect angle setting on their calculator.

e. This question was not well done throughout. Analytical approaches were almost always unsuccessful as a result of poor integration and differentiation skills and many of the errors were a result of having the GDC in degree mode. Few candidates attempted part (e). Of those that did, many attempted to find the largest local maximum of the graph rather than least local minimum as they did not recognise speed as $|v|$.

Let $f(x) = \mathrm{e}^x(1 - x^2)$.

Part of the graph of $y = f(x)$, for $-6 \le x \le 2$, is shown below. The x-coordinates of the local minimum and maximum points are r and s respectively.



a. Show that $f'(x) = \mathrm{e}^x(1 - 2x - x^2)$. [3]

b. Write down the **equation** of the horizontal asymptote. [1]

c. Write down the value of *r* and of *s*. [4]

d. Let *L* be the normal to the curve of *f* at $\mathrm{P}(0, 1)$. Show that *L* has equation $x + y = 1$. [4]

e(i) and (ii). Let *R* be the region enclosed by the curve $y = f(x)$ and the line *L*. [5]

(i) Find an expression for the area of *R*.

(ii) Calculate the area of *R*.

Markscheme

a. evidence of using the product rule ***M1***

$f'(x) = \mathrm{e}^x(1 - x^2) + \mathrm{e}^x(-2x)$ ***A1A1***

Note: Award ***A1*** for $\mathrm{e}^x(1 - x^2)$, ***A1*** for $\mathrm{e}^x(-2x)$.

$f'(x) = \mathrm{e}^x(1 - 2x - x^2)$ ***AG*** ***N0***

[3 marks]

b. $y = 0$ ***A1*** ***N1***

[1 mark]

c. at the local maximum or minimum point

$f'(x) = 0$ $(\mathrm{e}^x(1 - 2x - x^2) = 0)$ ***(M1)***

$\Rightarrow 1 - 2x - x^2 = 0$ ***(M1)***

$r = -2.41$ $s = 0.414$ ***A1A1*** ***N2N2***

[4 marks]

d. $f'(0) = 1$ ***A1***

gradient of the normal $= -1$ ***A1***

evidence of substituting into an equation for a straight line ***(M1)***

correct substitution ***A1***

e.g. $y - 1 = -1(x - 0)$, $y - 1 = -x$, $y = -x + 1$

$x + y = 1$ ***AG*** ***N0***

[4 marks]

e(i) and (ii). (i) intersection points at $x = 0$ and $x = 1$ (may be seen as the limits) ***(A1)***

approach involving subtraction and integrals ***(M1)***

fully correct expression ***A2*** ***N4***

e.g. $\int_0^1 \left(\mathrm{e}^x(1 - x^2) - (1 - x)\right)\mathrm{d}x$, $\int_0^1 f(x)\mathrm{d}x - \int_0^1 (1 - x)\mathrm{d}x$

(ii) area $R = 0.5$ ***A1*** ***N1***

[5 marks]

Examiners report

a. Many candidates clearly applied the product rule to correctly show the given derivative. Some candidates missed the multiplicative nature of the function and attempted to apply a chain rule instead.

b. For part (b), the equation of the horizontal asymptote was commonly written as $x = 0$.

c. Although part (c) was a "write down" question where no working is required, a good number of candidates used an algebraic method of solving for *r* and *s* which sometimes returned incorrect answers. Those who used their GDC usually found correct values, although not always to three significant figures.

d. In part (d), many candidates showed some skill showing the equation of a normal, although some tried to work with the gradient of the tangent.

e(i) and (ii). Surprisingly few candidates set up a completely correct expression for the area between curves that considered both integration and the correct subtraction of functions. Using limits of -6 and 2 was a common error, as was integrating on $f(x)$ alone. Where candidates did write a correct expression, many attempted to perform analytic techniques to calculate the area instead of using their GDC.

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

a. Show that $f(x) = \log_3 2x$. [2]

b. Find the value of $f(0.5)$ and of $f(4.5)$. [3]

c(i), (ii) and (iii). The function *f* can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$. [6]

(i) Write down the value of *a* and of *b* .

(ii) Hence on graph paper, **sketch** the graph of *f* , for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

d. Write down the value of $f^{-1}(0)$. [1]

e. The point A lies on the graph of *f* . At A, $x = 4.5$. [4]

On your diagram, sketch the graph of f^{-1} , noting clearly the image of point A.

Markscheme

a. combining 2 terms ***(A1)***

e.g. $\log_3 8x - \log_3 4$, $\log_3 \frac{1}{2}x + \log_3 4$

expression which clearly leads to answer given ***A1***

e.g. $\log_3 \frac{8x}{4}$, $\log_3 \frac{4x}{2}$

$f(x) = \log_3 2x$ ***AG*** ***N0***

[2 marks]

b. attempt to substitute either value into f ***(M1)***

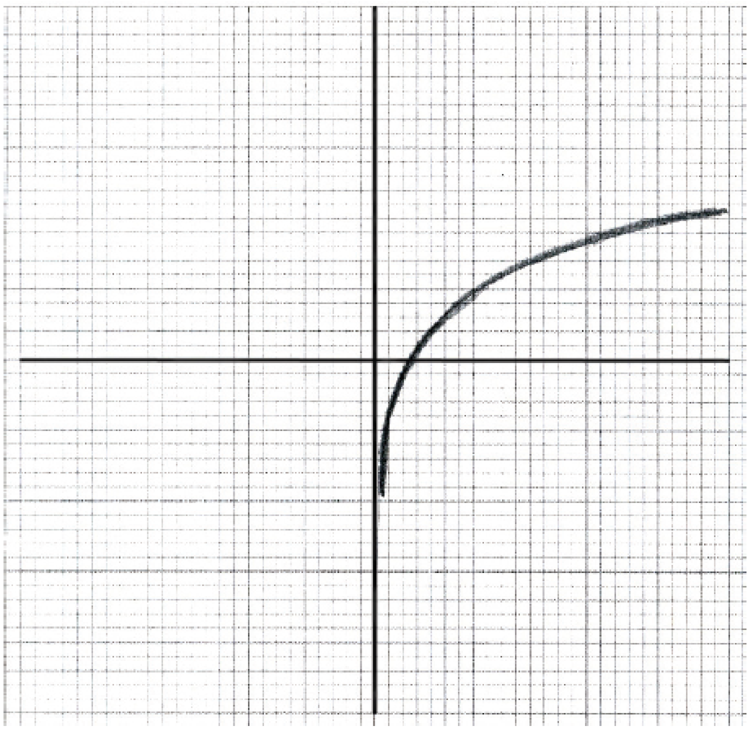
e.g. $\log_3 1$, $\log_3 9$

$f(0.5) = 0$, $f(4.5) = 2$ ***A1A1*** ***N3***

[3 marks]

c(i), (ii) and (iii). (i) $a = 2$, $b = 3$ ***A1A1*** ***N1N1***

(ii)



A1A1A1 ***N3***

Note: Award ***A1*** for sketch approximately through $(0.5 \pm 0.1, 0 \pm 0.1)$, ***A1*** for approximately correct shape, ***A1*** for sketch asymptotic to the y-axis.

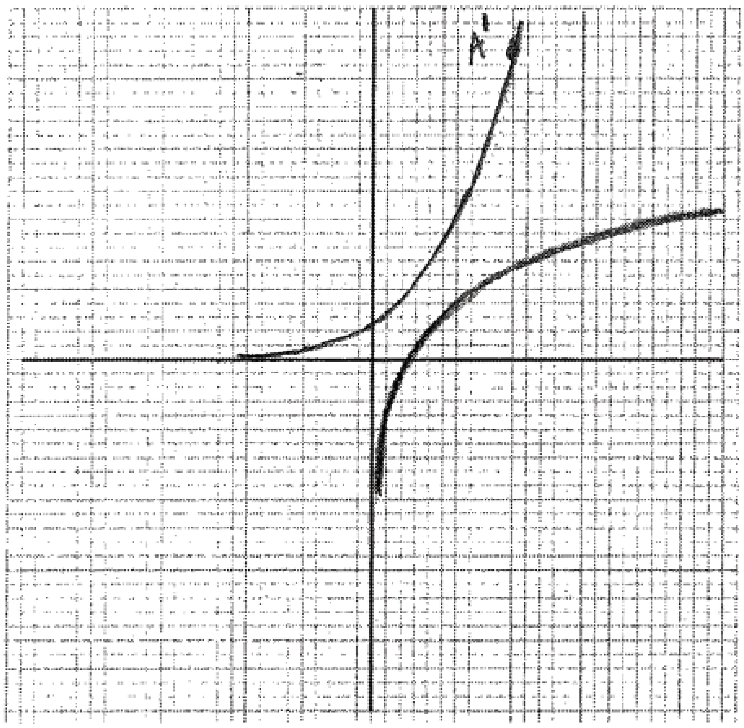
(iii) $x = 0$ (must be an equation) ***A1*** ***N1***

[6 marks]

d. $f^{-1}(0) = 0.5$ ***A1*** ***N1***

[1 mark]

e.



A1A1A1A1 *N4*

Note: Award ***A1*** for sketch approximately through $(0 \pm 0.1, 0.5 \pm 0.1)$, ***A1*** for approximately correct shape of the graph reflected over $y = x$, ***A1*** for sketch asymptotic to *x*-axis, ***A1*** for point $(2 \pm 0.1, 4.5 \pm 0.1)$ clearly marked and on curve.

[4 marks]

Examiners report

a. Few candidates had difficulty with part (a) although it was often communicated using some very sloppy applications of the rules of logarithm, writing $\frac{\log 16}{\log 4}$ instead of $\log\left(\frac{16}{4}\right)$.

b. Part (b) was generally done well.

c(i), (ii) and (iii). Part (c) (i) was generally done well; candidates seemed quite comfortable changing bases. There were some very good sketches in (c) (ii), but there were also some very poor ones with candidates only considering shape and not the location of the *x*-intercept or the asymptote. A surprising number of candidates did not use the scale required by the question and/or did not use graph paper to sketch the graph. In some cases, it was evident that students simply transposed their graphs from their GDC without any analytical consideration.

d. Part (d) was poorly done as candidates did not consider the command term, “write down” and often proceeded to find the inverse function before making the appropriate substitution.

e. Part (e) eluded a great many candidates as most preferred to attempt to find the inverse analytically rather than simply reflecting the graph of *f* in the line $y = x$. This graph also suffered from the same sort of problems as the graph in (c) (ii). Some students did not have their curve passing through $(2, 4.5)$ nor did they clearly indicate its position as instructed. This point was often mislabelled on the graph of *f*. The efforts in this question demonstrated that students often work tenuously from one question to the next, without considering the "big picture", thereby failing to make important links with earlier parts of the question.

Let $f(x) = 2x^2 + 4x - 6$.

a. Express $f(x)$ in the form $f(x) = 2(x-h)^2 + k$. [3]

b. Write down the equation of the axis of symmetry of the graph of f . [1]

c. Express $f(x)$ in the form $f(x) = 2(x-p)(x-q)$. [2]

Markscheme

a. evidence of obtaining the vertex ***(M1)***

e.g. a graph, $x = -\frac{b}{2a}$, completing the square

$f(x) = 2(x+1)^2 - 8$ ***A2 N3***

[3 marks]

b. $x = -1$ (equation must be seen) ***A1 N1***

[1 mark]

c. $f(x) = 2(x-1)(x+3)$ ***A1A1 N2***

[2 marks]

Examiners report

a. Many candidates answered this question with great ease. Still, some found themselves unable to correctly find the vertex algebraically, often mixing the signs of the *h* and *k* values. Using the GDC may have been a more fruitful approach. Some candidates did not write the axis of symmetry as an equation.

b. Many candidates answered this question with great ease. Still, some found themselves unable to correctly find the vertex algebraically, often mixing the signs of the *h* and *k* values. Using the GDC may have been a more fruitful approach. Some candidates did not write the axis of symmetry as an equation.

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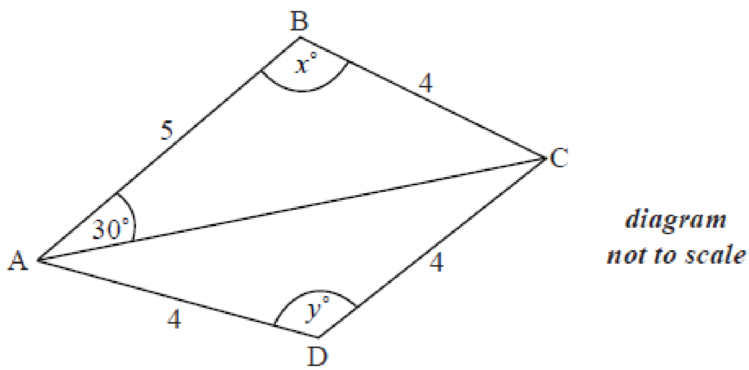
The diagram below shows a quadrilateral ABCD with obtuse angles $\mathrm{A\widehat{B}C}$ and $\mathrm{A\widehat{D}C}$.



AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm , $\mathrm{B\widehat{A}C} = 30^\circ$, $\mathrm{A\widehat{B}C} = x^\circ$, $\mathrm{A\widehat{D}C} = y^\circ$.

a. Use the cosine rule to show that $\mathrm{AC} = \sqrt{41 - 40\cos x}$. [1]

b. Use the sine rule in triangle ABC to find another expression for AC. [2]

c. (i) Hence, find x, giving your answer to two decimal places. [6]

(ii) Find AC .

d(i) and (ii). (i) Find y. [5]

(ii) Hence, or otherwise, find the area of triangle ACD.

Markscheme

a. correct substitution ***A1***

e.g. $25 + 16 - 40\cos x$, $5^2 + 4^2 - 2 \times 4 \times 5\cos x$

$\mathrm{AC} = \sqrt{41 - 40\cos x}$ ***AG***

[1 mark]

b. correct substitution ***A1***

e.g. $\frac{\mathrm{AC}}{\sin x} = \frac{4}{\sin 30}$, $\frac{1}{2}\mathrm{AC} = 4\sin x$

$\mathrm{AC} = 8\sin x$ (accept $\frac{4\sin x}{\sin 30}$) ***A1 N1***

[2 marks]

c. (i) evidence of appropriate approach using AC ***M1***

e.g. $8\sin x = \sqrt{41 - 40\cos x}$, sketch showing intersection

correct solution $8.682\ldots$, $111.317\ldots$ ***(A1)***

obtuse value $111.317\ldots$ ***(A1)***

$x = 111.32$ to 2 dp (do **not** accept the radian answer 1.94) ***A1 N2***

(ii) substituting value of x into either expression for AC ***(M1)***

e.g. $\mathrm{AC} = 8\sin 111.32$

$\mathrm{AC} = 7.45$ ***A1 N2***

[6 marks]

d(i) and (ii). (i) evidence of choosing cosine rule ***(M1)*** e.g. $\cos B = \frac{a^2+c^2-b^2}{2ac}$

correct substitution ***A1***

e.g. $\frac{4^2+4^2-7.45^2}{2\times4\times4}$, $7.45^2 = 32 - 32\cos y$, $\cos y = -0.734\ldots$

$y = 137$ ***A1*** ***N2***

(ii) correct substitution into area formula ***(A1)***

e.g. $\frac{1}{2}\times 4\times 4\times \sin 137$, $8\sin 137$

area $= 5.42$ ***A1*** ***N2***

[5 marks]

Examiners report

a. Many candidates worked comfortably with the sine and cosine rules in part (a) and (b).

b. Many candidates worked comfortably with the sine and cosine rules in part (a) and (b). Equally as many did not take the cue from the word "hence" and used an alternate method to solve the problem and thus did not receive full marks. Those who managed to set up an equation, again did not go directly to their GDC but rather engaged in a long, laborious analytical approach that was usually unsuccessful.

c. Equally as many did not take the cue from the word "hence" and used an alternate method to solve the problem and thus did not receive full marks. Those who managed to set up an equation, again did not go directly to their GDC but rather engaged in a long, laborious analytical approach that was usually unsuccessful. No matter what values were found in (c) (i) most candidates recovered and earned follow through marks for the remainder of the question. A large number of candidates worked in the wrong mode and rounded prematurely throughout this question often resulting in accuracy penalties.

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Let $h(x) = \frac{2x-1}{x+1}$, $x \neq -1$.

a. Find $h^{-1}(x)$. [4]

b(i), (ii) and (iii) (i) Sketch the graph of h for $-4 \le x \le 4$ and $-5 \le y \le 8$, including any asymptotes. [7]

(ii) Write down the equations of the asymptotes.

(iii) Write down the x-intercept of the graph of h .

c(i) and (ii) Let R be the region in the first quadrant enclosed by the graph of h , the x-axis and the line $x = 3$. [5]

(i) Find the area of R.

(ii) Write down an expression for the volume obtained when R is revolved through 360° about the x-axis.

Markscheme

a. $y = \frac{2x-1}{x+1}$

interchanging x and y (seen anywhere) ***M1***

e.g. $x = \frac{2y-1}{y+1}$

correct working ***A1***

e.g. $xy + x = 2y - 1$

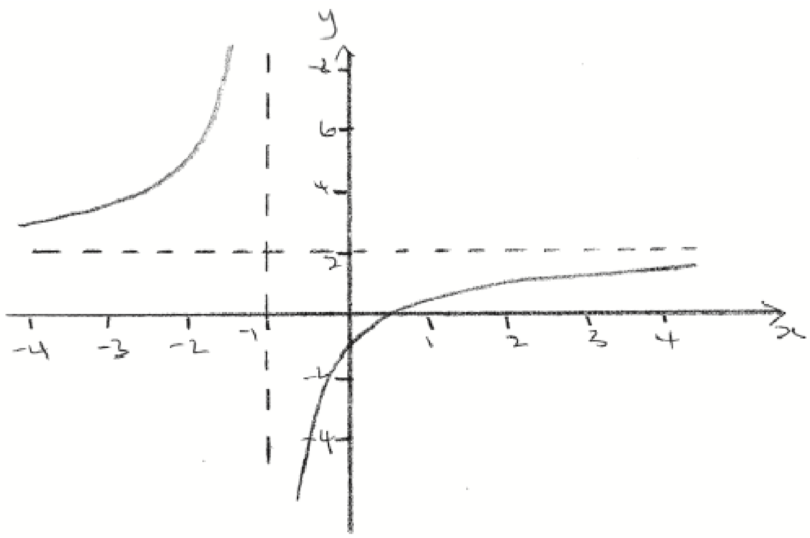
collecting terms ***A1***

e.g. $x + 1 = 2y - xy$, $x + 1 = y(2 - x)$

$h^{-1}(x) = \frac{x+1}{2-x}$ ***A1*** ***N2***

[4 marks]

b(i), (ii) and (iii).



A1A1A1A1 ***N4***

Note: Award ***A1*** for approximately correct intercepts, ***A1*** for correct shape, ***A1*** for asymptotes, ***A1*** for approximately correct domain and range.

(ii) $x = -1$, $y = 2$ ***A1A1*** ***N2***

(iii) $\frac{1}{2}$ ***A1*** ***N1***

[7 marks]

c(i) and (ii).

(i) area $= 2.06$ ***A2*** ***N2***

(ii) attempt to substitute into volume formula (do not accept $\pi \int_a^b y^2 \mathrm{d}x$) ***M1***

volume $= \pi \int_{\frac{1}{2}}^{3} \left(\frac{2x-1}{x+1}\right)^2 \mathrm{d}x$ ***A2*** ***N3***

[5 marks]

Examiners report

a. [N/A]

b(i), (ii) and (iii). [N/A]

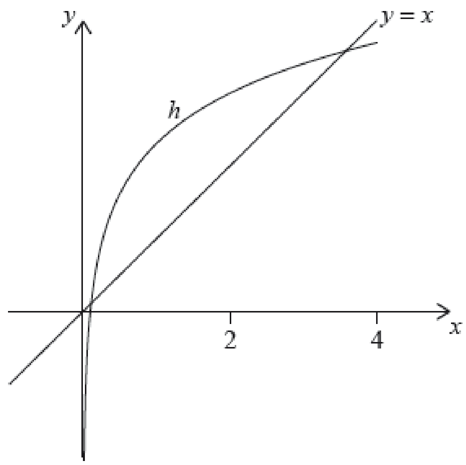
c(i) and (ii). [N/A]

Let $f(x) = \ln x$ and $g(x) = 3 + \ln\left(\frac{x}{2}\right)$, for $x > 0$.

The graph of g can be obtained from the graph of f by two transformations:

$$\text{a horizontal stretch of scale factor } q \text{ followed by}$$
$$\text{a translation of } \begin{pmatrix} h \\ k \end{pmatrix}.$$

Let $h(x) = g(x) \times \cos(0.1x)$, for $0 < x < 4$. The following diagram shows the graph of h and the line $y = x$.



The graph of h intersects the graph of h^{-1} at two points. These points have x coordinates 0.111 and 3.31 correct to three significant figures.

a.i. Write down the value of q; [1]

a.ii. Write down the value of h; [1]

a.iii. Write down the value of k. [1]

b.i. Find $\int_{0.111}^{3.31} (h(x) - x)\,\mathrm{d}x$. [2]

b.ii. Hence, find the area of the region enclosed by the graphs of h and h^{-1}. [3]

c. Let d be the vertical distance from a point on the graph of h to the line $y = x$. There is a point $\mathrm{P}(a,\ b)$ on the graph of h where d is a maximum. [7]

Find the coordinates of P, where $0.111 < a < 3.31$.

Markscheme

a.i. $q = 2$ ***A1*** ***N1***

Note: Accept $q = 1$, $h = 0$, and $k = 3 - \ln(2)$, 2.31 as candidate may have rewritten $g(x)$ as equal to $3 + \ln(x) - \ln(2)$.

[1 mark]

a.ii. $h = 0$ ***A1*** ***N1***

Note: Accept $q = 1$, $h = 0$, and $k = 3 - \ln(2)$, 2.31 as candidate may have rewritten $g(x)$ as equal to $3 + \ln(x) - \ln(2)$.

[1 mark]

a.iii. $k = 3$ ***A1*** ***N1***

Note: Accept $q = 1$, $h = 0$, and $k = 3 - \ln(2)$, 2.31 as candidate may have rewritten $g(x)$ as equal to $3 + \ln(x) - \ln(2)$.

[1 mark]

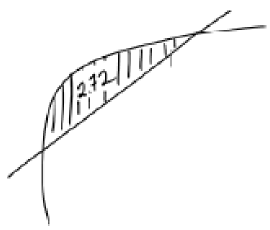
b.i. 2.72409

2.72 ***A2*** ***N2***

[2 marks]

b.ii. recognizing area between $y = x$ and h equals 2.72 ***(M1)***

eg



recognizing graphs of h and h^{-1} are reflections of each other in $y = x$ ***(M1)***

eg area between $y = x$ and h equals between $y = x$ and h^{-1}

$2 \times 2.72 \int_{0.111}^{3.31} \left(x - h^{-1}(x)\right) \mathrm{d}x = 2.72$

5.44819

5.45 ***A1*** ***N3***

[??? marks]

c. valid attempt to find d ***(M1)***

eg difference in y-coordinates, $d = h(x) - x$

correct expression for d ***(A1)***

eg $\left(\ln \frac{1}{2}x + 3\right)(\cos 0.1x) - x$

valid approach to find when d is a maximum ***(M1)***

eg max on sketch of d, attempt to solve $d' = 0$

0.973679

$x = 0.974$ ***A2*** ***N4***

substituting **their** x value into $h(x)$ ***(M1)***

2.26938

$y = 2.27$ ***A1*** ***N2***

[7 marks]

Examiners report

a.i. [N/A]

a.ii. [N/A]

a.iii. [N/A]

[N/A]

b.ii. [N/A]
c. [N/A]

Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

a. Sketch the graph of f . [3]

b. Write down [3]

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

c. Hence write $f(x)$ in the form $p\sin(qx + r)$. [3]

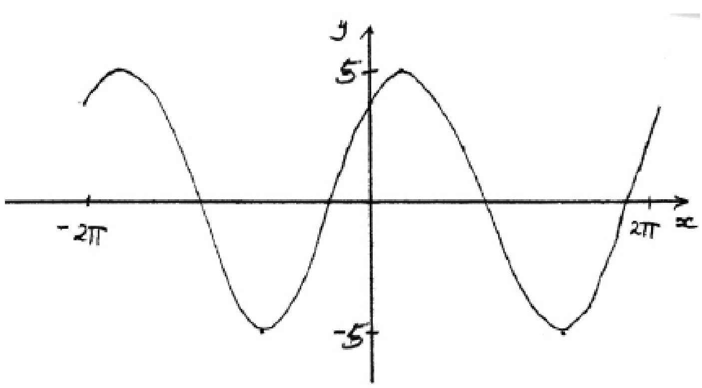
d. Write down one value of x such that $f'(x) = 0$. [2]

e. Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions. [2]

f. Let $g(x) = \ln(x + 1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x. [5]

Markscheme

a.



A1A1A1 *N3*

Note: Award *A1* for approximately sinusoidal shape, *A1* for end points approximately correct $(-2\pi, 4)$ $(2\pi, 4)$, *A1* for approximately correct position of graph, (y-intercept $(0, 4)$, maximum to right of y-axis).

[3 marks]

b. (i) 5 *A1* *N1*

(ii) 2π (6.28) *A1* *N1*

(iii) -0.927 *A1* *N1*

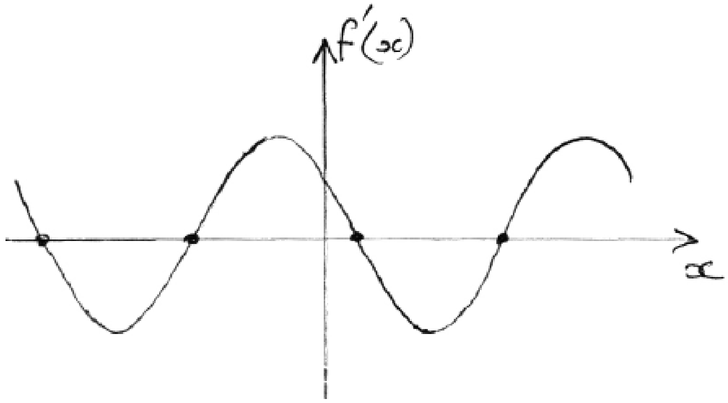
[3 marks]

c. $f(x) = 5\sin(x + 0.927)$ (accept $p = 5$, $q = 1$, $r = 0.927$) ***A1A1A1*** ***N3***

[3 marks]

d. evidence of correct approach ***(M1)***

e.g. max/min, sketch of $f'(x)$ indicating roots



one 3 s.f. value which rounds to one of -5.6, -2.5, 0.64, 3.8 ***A1*** ***N2***

[2 marks]

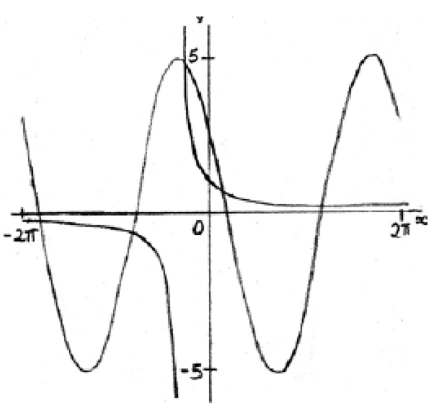
e. $k = -5$, $k = 5$ ***A1A1*** ***N2***

[2 marks]

f. **METHOD 1**

graphical approach (but must involve derivative functions) ***M1***

e.g.



each curve ***A1A1***

$x = 0.511$ ***A2*** ***N2***

METHOD 2

$g'(x) = \frac{1}{x+1}$ ***A1***

$f'(x) = 3\cos x - 4\sin x$ $(5\cos(x + 0.927))$ ***A1***

evidence of attempt to solve $g'(x) = f'(x)$ ***M1***

$x = 0.511$ ***A2*** ***N2***

[5 marks]

Examiners report

a. Some graphs in part (a) were almost too detailed for just a sketch but more often, the important features were far from clear. Some graphs lacked scales on the axes.

b. A number of candidates had difficulty finding the period in part (b)(ii).

c. A number of candidates had difficulty writing the correct value of q in part (c).

d. The most common approach in part (d) was to differentiate and set $f'(x) = 0$. Fewer students found the values of x given by the maximum or minimum values on their graphs.

e. Part (e) proved challenging for many candidates, although if candidates answered this part, they generally did so correctly.

f. In part (f), many candidates were able to get as far as equating the two derivatives but fewer used their GDC to solve the resulting equation. Again, many had trouble demonstrating their method of solution.

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

a. Write down the coordinates of the vertex of the graph of g . [2]

b. Express g in the form $g(x) = 3(x - p)^2 + q$. [2]

c. The graph of h is the reflection of the graph of g in the x-axis. [2]

Write down the coordinates of the vertex of the graph of h .

Markscheme

a. $(1,\ -2)$ ***A1A1 N2***

[2 marks]

b. $g(x) = 3(x - 1)^2 - 2$ (accept $p = 1$, $q = -2$) ***A1A1 N2***

[2 marks]

c. $(1, 2)$ ***A1A1 N2***

[2 marks]

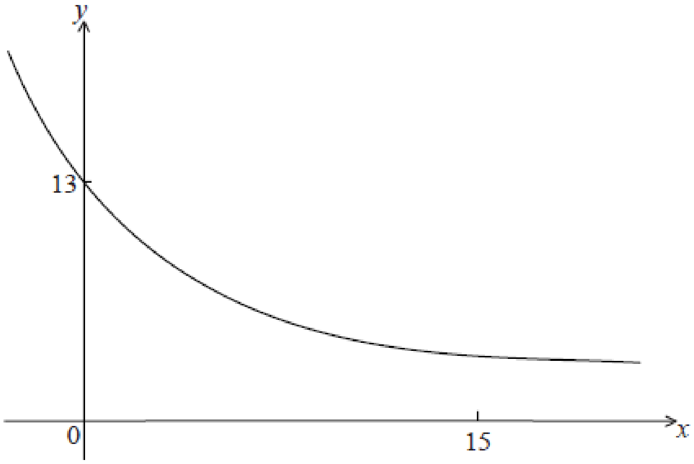
Examiners report

a. Most candidates had little difficulty with this question.

b. Most candidates had little difficulty with this question.

c. Most candidates had little difficulty with this question. In part (c), a few reflected the vertex in the y-axis rather than the x-axis.

Let $f(x) = Ae^{kx} + 3$. Part of the graph of f is shown below.



The y-intercept is at (0, 13) .

a. Show that $A = 10$. [2]

b. Given that $f(15) = 3.49$ (correct to 3 significant figures), find the value of k. [3]

c(i), (ii) and (iii). (i) Using your value of k , find $f'(x)$. [5]

(ii) Hence, explain why f is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph f .

d. Let $g(x) = -x^2 + 12x - 24$. [6]

Find the area enclosed by the graphs of f and g .

Markscheme

a. substituting (0, 13) into function ***M1***

e.g. $13 = Ae^0 + 3$

$13 = A + 3$ ***A1***

$A = 10$ ***AG*** ***N0***

[2 marks]

b. substituting into $f(15) = 3.49$ ***A1***

e.g. $3.49 = 10e^{15k} + 3$, $0.049 = e^{15k}$

evidence of solving equation ***(M1)***

e.g. sketch, using ln

$k = -0.201$ (accept $\frac{\ln 0.049}{15}$) ***A1*** ***N2***

[3 marks]

c(i), (ii) and (iii). (i) $f(x) = 10\mathrm{e}^{-0.201x} + 3$

$f(x) = 10\mathrm{e}^{-0.201x} \times -0.201 \ (= -2.01\mathrm{e}^{-0.201x})$ ***A1A1A1*** ***N3***

Note: Award ***A1*** for $10\mathrm{e}^{-0.201x}$, ***A1*** for $\times - 0.201$, ***A1*** for the derivative of 3 is zero.

(ii) valid reason with reference to derivative ***R1*** ***N1***

e.g. $f'(x) < 0$, derivative always negative

(iii) $y = 3$ ***A1*** ***N1***

[5 marks]

d. finding limits $3.8953\ldots$, $8.6940\ldots$ (seen anywhere) ***A1A1***

evidence of integrating and subtracting functions ***(M1)***

correct expression ***A1***

e.g. $\int_{3.90}^{8.69} g(x) - f(x)\mathrm{d}x$, $\int_{3.90}^{8.69} \left[\left(-x^2 + 12x - 24\right) - \left(10\mathrm{e}^{-0.201x} + 3\right)\right]\mathrm{d}x$

area $= 19.5$ ***A2*** ***N4***

[6 marks]

Examiners report

a. This question was quite well done by a great number of candidates indicating that calculus is a topic that is covered well by most centres. Parts (a) and (b) proved very accessible to many candidates.

b. This question was quite well done by a great number of candidates indicating that calculus is a topic that is covered well by most centres. Parts (a) and (b) proved very accessible to many candidates.

c(i), (ii) and (iii). The chain rule in part (c) was also carried out well. Few however, recognized the command term “hence” and that $f'(x) < 0$ guarantees a decreasing function. A common answer for the equation of the asymptote was to give $y = 0$ or $x = 3$.

d. In part (d), it was again surprising and somewhat disappointing to see how few candidates were able to use their GDC effectively to find the area between curves, often not finding correct limits, and often trying to evaluate the definite integral without the GDC, which led nowhere.

Let $f(x) = \frac{20x}{\mathrm{e}^{0.3x}}$, for $0 \le x \le 20$.

a. Sketch the graph of *f* . [3]

b(i) and (ii). (i) Write down the *x*-coordinate of the maximum point on the graph of *f* . [3]

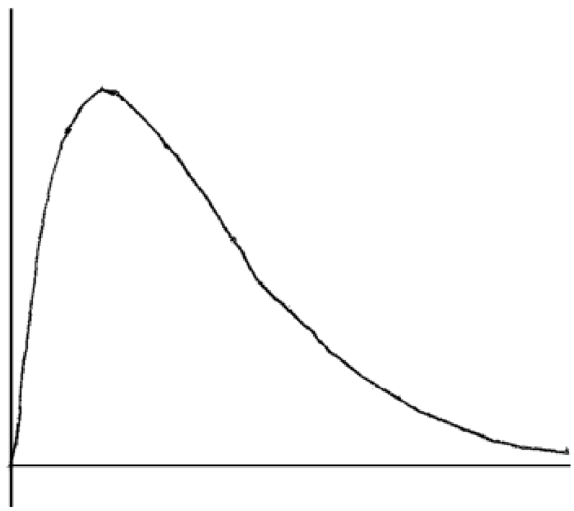
(ii) Write down the interval where *f* is increasing.

c. Show that $f'(x) = \frac{20-6x}{\mathrm{e}^{0.3x}}$. [5]

d. Find the interval where the rate of change of *f* is increasing. [4]

Markscheme

a.



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape with inflexion/change of curvature, ***A1*** for maximum skewed to the left, ***A1*** for asymptotic behaviour to the right.

[3 marks]

b(i) and (ii). (i) $x = 3.33$ ***A1*** ***N1***

(ii) correct interval, with right end point $3\frac{1}{3}$ ***A1A1*** ***N2***

e.g. $0 < x \leq 3.33$, $0 \leq x < 3\frac{1}{3}$

Note: Accept any inequalities in the right direction.

[3 marks]

c. valid approach ***(M1)***

e.g. quotient rule, product rule

2 correct derivatives (must be seen in product or quotient rule) ***(A1)(A1)***

e.g. 20 , $0.3\mathrm{e}^{0.3x}$ or $-0.3\mathrm{e}^{-0.3x}$

correct substitution into product or quotient rule ***A1***

e.g. $\frac{20\mathrm{e}^{0.3x}-20x(0.3)\mathrm{e}^{0.3x}}{(\mathrm{e}^{0.3x})^2}$, $20\mathrm{e}^{-0.3x}+20x(-0.3)\mathrm{e}^{-0.3x}$

correct working ***A1***

e.g. $\frac{20\mathrm{e}^{0.3x}-6x\mathrm{e}^{0.3x}}{\mathrm{e}^{0.6x}}$, $\frac{\mathrm{e}^{0.3x}(20-20x(0.3))}{(\mathrm{e}^{0.3x})^2}$, $\mathrm{e}^{-0.3x}(20+20x(-0.3))$

$f'(x) = \frac{20-6x}{\mathrm{e}^{0.3x}}$ ***AG*** ***N0***

[5 marks]

d. consideration of f' or f'' ***(M1)***

valid reasoning ***R1***

e.g. sketch of f' , f'' is positive, $f''=0$, reference to minimum of f'

correct value $6.6666666\ldots \left(6\frac{2}{3}\right)$ ***(A1)***

correct interval, with **both** endpoints ***A1*** ***N3***

e.g. $6.67 < x \leq 20$, $6\frac{2}{3} \leq x < 20$

[4 marks]

Examiners report

a. Many candidates earned the first four marks of the question in parts (a) and (b) for correctly using their GDC to graph and find the maximum value.

b(i) and (ii). Many candidates earned the first four marks of the question in parts (a) and (b) for correctly using their GDC to graph and find the maximum value.

c. Most had a valid approach in part (c) using either the quotient or product rule, but many had difficulty applying the chain rule with a function involving e and simplifying.

d. Part (d) was difficult for most candidates. Although many associated rate of change with derivative, only the best-prepared students had valid reasoning and could find the correct interval with both endpoints.

Let $f(x) = \mathrm{e}^{\frac{x}{4}}$ and $g(x) = mx$, where $m \geq 0$, and $-5 \leq x \leq 5$. Let R be the region enclosed by the y-axis, the graph of f , and the graph of g .

Let $m = 1$.

a. (i) Sketch the graphs of f and g on the same axes. [7]

(ii) Find the area of R .

a.ii. Find the area of R . [5]

b. Consider all values of m such that the graphs of f and g intersect. Find the value of m that gives the greatest value for the area of R . [8]

Markscheme

a. ***(i)***

A1A1 N2

Notes: Award ***A1*** for the graph of f positive, increasing and concave up.

Award ***A1*** for graph of g increasing and linear with y-intercept of 0.

Penalize one mark if domain is not $[-5, 5]$ and/or if f and g do not intersect in the first quadrant.

[2 marks]

(ii)

attempt to find intersection of the graphs of f ***and*** g ***(M1)***

eg $\mathrm{e}^{\frac{x}{4}} = x$

$x = 1.42961\ldots$ ***A1***

valid attempt to find area of R ***(M1)***

eg $\int (x - e^{\frac{x}{4}})dx$ **,** $\int_0^1 (g - f)$ **,** $\int (f - g)$

area $= 0.697$ ***A2*** ***N3***

[5 marks]

a.ii. attempt to find intersection of the graphs of f and g ***(M1)***

eg $e^{\frac{x}{4}} = x$

$x = 1.42961\ldots$ ***A1***

valid attempt to find area of R ***(M1)***

eg $\int (x - e^{\frac{x}{4}})dx$, $\int_0^1 (g - f)$, $\int (f - g)$

area $= 0.697$ ***A2*** ***N3***

[5 marks]

b. recognize that area of R is a maximum at point of tangency ***(R1)***

eg $m = f'(x)$

equating functions ***(M1)***

eg $f(x) = g(x)$, $e^{\frac{x}{4}} = mx$

$f'(x) = \frac{1}{4}e^{\frac{x}{4}}$ ***(A1)***

equating gradients ***(A1)***

eg $f'(x) = g'(x)$, $\frac{1}{4}e^{\frac{x}{4}} = m$

attempt to solve system of two equations for x ***(M1)***

eg $\frac{1}{4}e^{\frac{x}{4}} \times x = e^{\frac{x}{4}}$

$x = 4$ ***(A1)***

attempt to find m ***(M1)***

eg $f'(4)$, $\frac{1}{4}e^{\frac{4}{4}}$

$m = \frac{1}{4}e$ (exact), 0.680 ***A1*** ***N3***

[8 marks]

Examiners report

a. There was a flaw with the domain noted in this question. While not an error in itself, it meant that part (b) no longer assessed what was intended. The markscheme included a variety of solutions based on candidate work seen, and examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were looked at during the grade award meeting.

While some candidates sketched accurate graphs on the given domain, the majority did not. Besides the common domain error, some exponential curves were graphed with several concavity changes.

a.ii. There was a flaw with the domain noted in this question. While not an error in itself, it meant that part (b) no longer assessed what was intended. The markscheme included a variety of solutions based on candidate work seen, and examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were looked at during the grade award meeting.

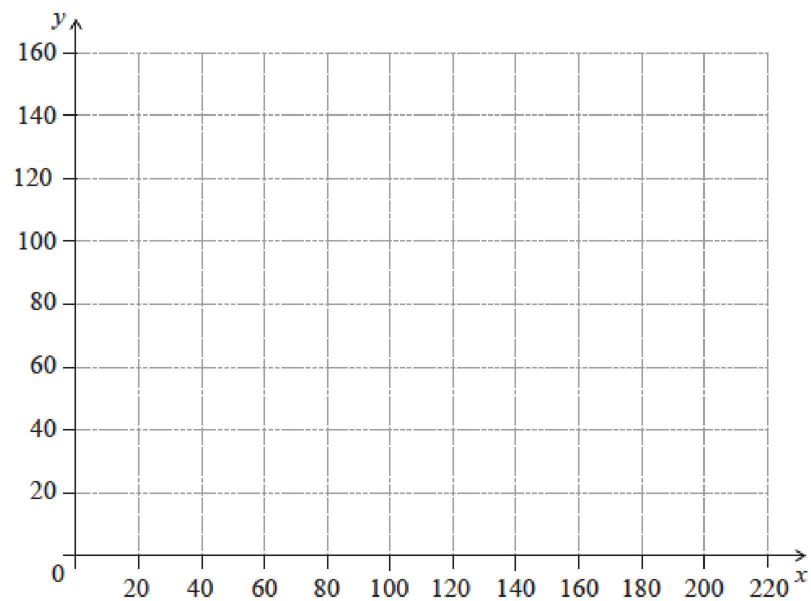
In part (a)(ii), most candidates found the intersection correctly. Those who used their GDC to evaluate the integral numerically were usually successful, unlike those who attempted to solve with antiderivatives. A common error was to find the area of the region enclosed by f and g (although it involved a point of intersection outside of the given domain), rather than the area of the region enclosed by f and g and the y-axis.

b. There was a flaw with the domain noted in this question. While not an error in itself, it meant that part (b) no longer assessed what was intended. The markscheme included a variety of solutions based on candidate work seen, and examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were looked at during the grade award meeting.

While some candidates were able to show some good reasoning in part (b), fewer were able to find the value of m which maximized the area of the region. In addition to the answer obtained from the restricted domain, full marks were awarded for the answer obtained by using the point of tangency.

Let $G(x) = 95\mathrm{e}^{(-0.02x)} + 40$, for $20 \leq x \leq 200$.

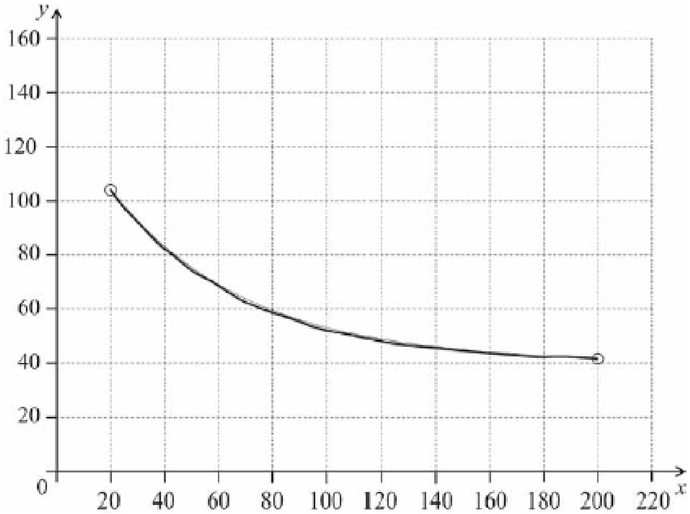
a. On the following grid, sketch the graph of G. [3]



b. Robin and Pat are planning a wedding banquet. The cost per guest, G dollars, is modelled by the function $G(n) = 95\mathrm{e}^{(-0.02n)} + 40$, for $20 \leq n \leq 200$, where n is the number of guests. [3]

Calculate the **total** cost for 45 guests.

Markscheme

a. 

A1A1A1 ***N3***

Note: Curve must be approximately correct exponential shape (concave up and decreasing). Only if the shape is approximately correct, award the following:

A1 for left endpoint in circle,

A1 for right endpoint in circle,

A1 for asymptotic to $y = 40$ (must not go below $y = 40$).

[3 marks]

b. attempt to find $G(45)$ ***(M1)***

eg 78.6241, value read from **their** graph

multiplying cost times number of people ***(M1)***

eg $45 \times 78.6241, \ G(45) \times 45$

3538.08

3540 (dollars) ***A1*** ***N2***

[3 marks]

Total [6 marks]

Examiners report

a. The majority of candidates were able to sketch the shape of the graph accurately, but graph sketching is an area of the syllabus in which candidates continue to lose marks. In this particular question, candidates often did not consider the given domain or failed to accurately show the behaviour of the graph close to the horizontal asymptote as $x \to \infty$.

b. In (b), most candidates were able to identify the initial approach by finding $G(45)$, but missed the fact that function defined the cost per guest and not the total cost.

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x - 5$, for $x \in \mathbb{R}$.

a. Find $f(8)$. [2]

b. Find $(g \circ f)(x)$. [2]

c. Solve $(g \circ f)(x) = 0$. [3]

Markscheme

a. attempt to substitute $x = 8$ ***(M1)***

eg $8^2 + 2 \times 8 + 1$

$f(8) = 81$ ***A1 N2***

[2 marks]

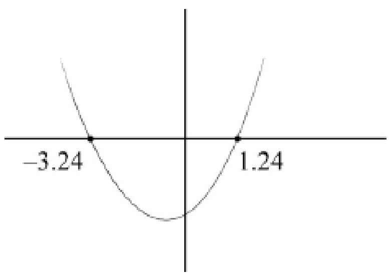
b. attempt to form composition (in any order) ***(M1)***

eg $f(x-5),\ g(f(x)),\ (x^2+2x+1)-5$

$(g \circ f)(x) = x^2 + 2x - 4$ ***A1 N2***

[2 marks]

c. valid approach ***(M1)***

eg $x = \frac{-2 \pm \sqrt{20}}{2}$, 

$1.23606,\ -3.23606$

$x = 1.24,\ x = -3.24$ ***A1A1 N3***

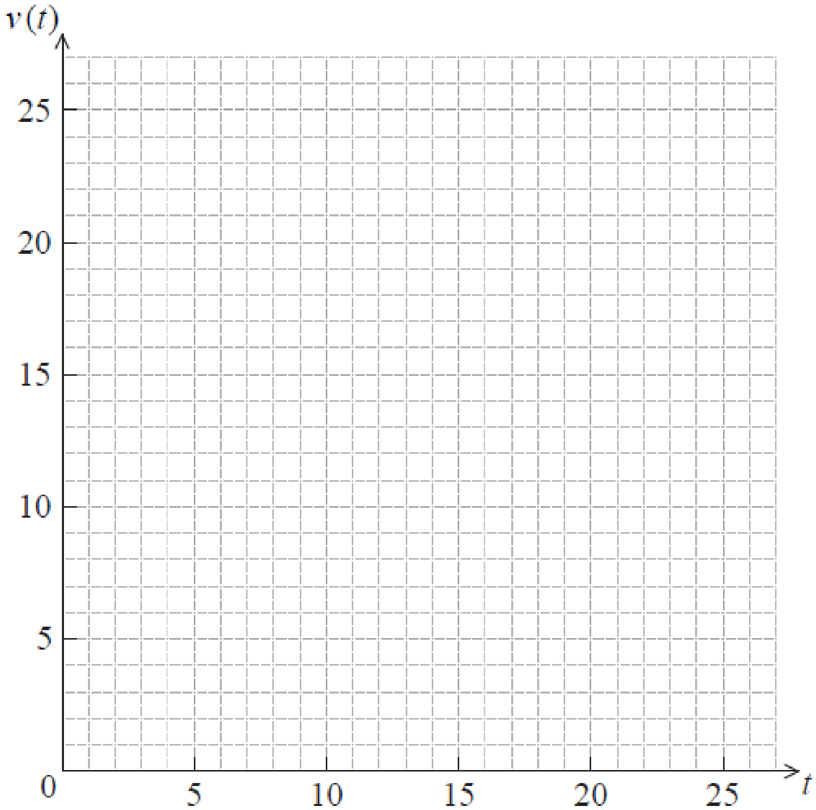
[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The velocity v ms^{-1} of an object after t seconds is given by $v(t) = 15\sqrt{t} - 3t$, for $0 \le t \le 25$.

a. On the grid below, sketch the graph of v , clearly indicating the maximum point. [3]

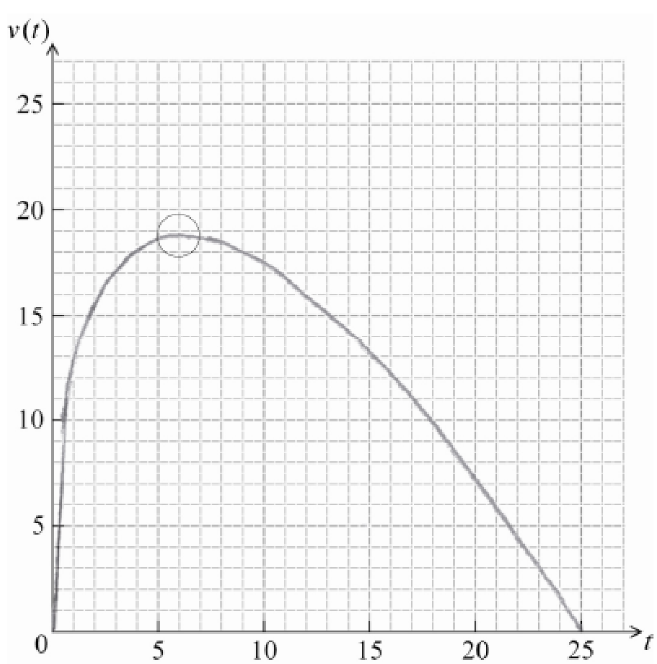


b(i) and (ii) (i) Write down an expression for d . [4]

(ii) Hence, write down the value of d .

Markscheme

a.



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape, ***A1*** for right endpoint at $(25, 0)$ and ***A1*** for maximum point in circle.

[3 marks]

b(i) and (ii) (i) recognizing that d is the area under the curve ***(M1)*** e.g. $\int v(t)$

correct expression in terms of t, with correct limits ***A2*** ***N3***

e.g. $d = \int_0^9 (15\sqrt{t} - 3t)\mathrm{d}t$, $d = \int_0^9 v\mathrm{d}t$

(ii) $d = 148.5$ (m) (accept 149 to 3 sf) ***A1*** ***N1***

[4 marks]

Examiners report

a. The graph in part (a) was well done. It was pleasing to see many candidates considering the domain as they sketched their graph.

b(i) and (ii). Part (b) (i) asked for an expression which bewildered a great many candidates. However, few had difficulty obtaining the correct answer in (b) (ii).

The first three terms of a geometric sequence are $u_1 = 0.64,\ u_2 = 1.6$, and $u_3 = 4$.

a. Find the value of r. [2]

b. Find the value of S_6. [2]

c. Find the least value of n such that $S_n > 75\,000$. [3]

Markscheme

a. valid approach ***(M1)***

eg $\frac{u_1}{u_2},\ \frac{4}{1.6},\ 1.6 = r(0.64)$

$r = 2.5 \quad \left(= \frac{5}{2}\right)$ ***A1*** ***N2***

[2 marks]

b. correct substitution into S_6 ***(A1)***

eg $\frac{0.64(2.5^6-1)}{2.5-1}$

$S_6 = 103.74$ (exact), 104 ***A1*** ***N2***

[2 marks]

c. **METHOD 1 (analytic)**

valid approach ***(M1)***

eg $\frac{0.64(2.5^n-1)}{2.5-1} > 75\,000,\ \frac{0.64(2.5^n-1)}{2.5-1} = 75\,000$

correct inequality (accept equation) ***(A1)***

eg $n > 13.1803,\ n = 13.2$

$n = 14$ ***A1*** ***N1***

METHOD 2 (table of values)

both crossover values ***A2***

eg $S_{13} = 63577.8,\ S_{14} = 158945$

$n = 14$ ***A1*** ***N1***

[3 marks]

Total [7 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following table shows a probability distribution for the random variable X, where $\mathrm{E}(X) = 1.2$.

x	0	1	2	3
$\mathrm{P}(X = x)$	p	$\frac{1}{2}$	$\frac{3}{10}$	q

A bag contains white and blue marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of blue marbles drawn is given by the random variable X.

A game is played in which three marbles are drawn from the bag of ten marbles, without replacement. A player wins a prize if three white marbles are drawn.

Jill plays the game nine times. Find the probability that she wins exactly two prizes.

Markscheme

valid approach ***(M1)***

eg $\mathrm{B}(n,\ p),\ \binom{n}{r} p^r q^{n-r},\ (0.167)^2(0.833)^7,\ \binom{9}{2}$

0.279081

0.279 ***A1*** ***N2***

[2 marks]

Examiners report

[N/A]

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n .$$

a(i) and (ii). (i) Find the number of taxis in the city at the end of 2005. [6]

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

b(i) and (ii). At the end of 2000 there were 25600 people in the city who used taxis. [6]

After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2560000}{10 + 90\mathrm{e}^{-0.1n}} .$$

(i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer.

c(i) and (ii). Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$. [5]

(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

Markscheme

a(i) and (ii). (i) $n = 5$ ***(A1)***

$T = 280 \times 1.12^5$

$T = 493$ ***A1 N2***

(ii) evidence of doubling ***(A1)***

e.g. 560

setting up equation ***A1***

e.g. $280 \times 1.12^n = 560$, $1.12^n = 2$

$n = 6.116\ldots$ ***(A1)***

in the year 2007 ***A1 N3***

[6 marks]

b(i) and (ii). (i) $P = \frac{2560000}{10+90\mathrm{e}^{-0.1(5)}}$ ***(A1)***

$P = 39635.993\ldots$ ***(A1)***

$P = 39636$ ***A1 N3***

(ii) $P = \frac{2560000}{10+90\mathrm{e}^{-0.1(7)}}$

$P = 46806.997\ldots$ ***A1***

not doubled ***A1 N0***

valid reason for **their** answer ***R1***

e.g. $P < 51200$

[6 marks]

c(i) and (ii). (i) correct value ***A2 N2*** e.g. $\frac{25600}{280}$, 91.4, 640 : 7

(ii) setting up an inequality (accept an equation, or reversed inequality) ***M1***

e.g. $\frac{P}{T} < 70$, $\frac{2560000}{(10+90e^{-0.1n})280\times1.12^{n}} < 70$

finding the value $9.31\ldots$ ***(A1)***

after 10 years ***A1*** ***N2***

[5 marks]

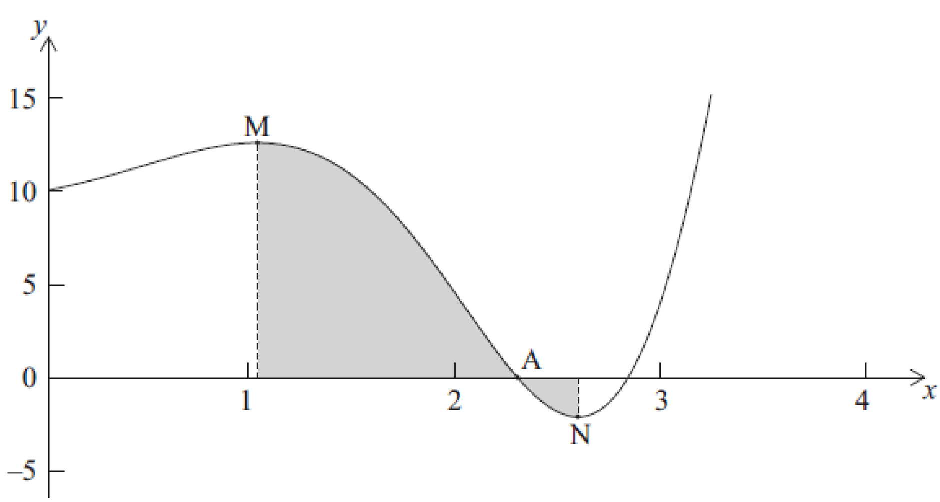
Examiners report

a(i) And (ii) A number of candidates found this question very accessible. In part (a), many correctly solved for *n*, but often incorrectly answered with the year 2006, thus misinterpreting that 6.12 years after the end of 2000 is in the year 2007.

b(i) and (ii) Many found correct values in part (b) and often justified their result by simply noting the value after seven years is less than 51200. A common alternative was to divide 46807 by 25600 and note that this ratio is less than two. There were still a good number of candidates who failed to provide any justification as instructed.

c(i) and (ii) Part (c) proved more challenging to candidates. Many found the correct ratio for *R*, however few candidates then created a proper equation or inequality by dividing the function for *P* by the function for *T* and setting this equal (or less) than 70. Such a function, although unfamiliar, can be solved using the graphing or solving features of the GDC. Many candidates chose a tabular approach but often only wrote down one value of the table, such as $n = 10$, $R = 68.3$. What is essential is to include the two values between which the correct answer falls. Sufficient evidence would include $n = 9$, $R = 70.8$ so that it is clear the value of $R = 70$ has been surpassed.

Let $f(x) = e^{x}\sin 2x + 10$, for $0 \le x \le 4$. Part of the graph of *f* is given below.



There is an *x*-intercept at the point A, a local maximum point at M, where $x = p$ and a local minimum point at N, where $x = q$.

a. Write down the *x*-coordinate of A. [1]

b(i) and (ii) Find the value of [2]

(i) p ;

(ii) q .

c. Find $\int_p^q f(x)\mathrm{d}x$. Explain why this is not the area of the shaded region. [3]

Markscheme

a. 2.31 ***A1*** ***N1***

[1 mark]

b(i) and (ii). (i) 1.02 ***A1*** ***N1***

(ii) 2.59 ***A1*** ***N1***

[2 marks]

c. $\int_p^q f(x)\mathrm{d}x = 9.96$ ***A1*** ***N1***

split into two regions, make the area below the *x*-axis positive ***R1R1*** ***N2***

[3 marks]

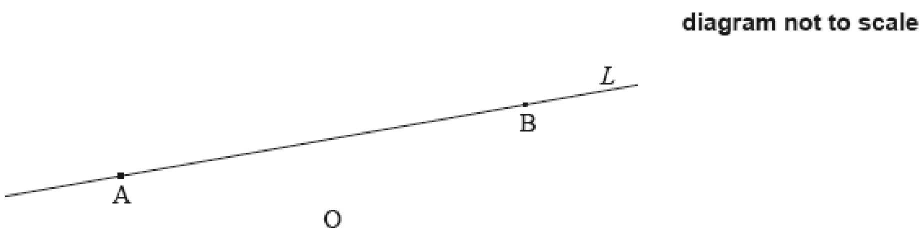
Examiners report

a. Parts (a) and (b) were generally well answered, the main problem being the accuracy.

b(i) and (ii). Parts (a) and (b) were generally well answered, the main problem being the accuracy.

c. Many students lacked the calculator skills to successfully complete (6)(c) in that they could not find the value of the definite integral. Some tried to find it by hand. When trying to explain why the integral was not the area, most knew the region under the *x*-axis was the cause of the integral not giving the total area, but the explanations were not sufficiently clear. It was often stated that the area below the axis was negative rather than the integral was negative.

The points A and B lie on a line L, and have position vectors $\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$ respectively. Let O be the origin. This is shown on the following diagram.



The point C also lies on L, such that $\overrightarrow{\mathrm{AC}} = 2\overrightarrow{\mathrm{CB}}$.

Let θ be the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{OC}}$.

Let D be a point such that $\overrightarrow{\mathrm{OD}} = k\overrightarrow{\mathrm{OC}}$, where $k > 1$. Let E be a point on L such that $\mathrm{C\hat{E}D}$ is a right angle. This is shown on the following diagram.

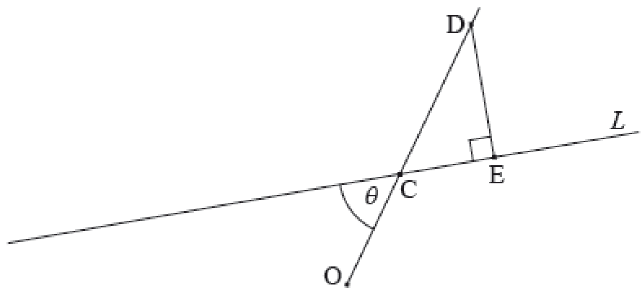


diagram not to scale

a. Find $\overrightarrow{\mathrm{AB}}$. [2]

b. Show that $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$. [[N/A

c. Find θ. [5]

d. (i) Show that $\left|\overrightarrow{\mathrm{DE}}\right| = (k-1)\left|\overrightarrow{\mathrm{OC}}\right|\sin\theta$. [6]

(ii) The distance from D to line L is less than 3 units. Find the possible values of k.

Markscheme

a. valid approach (addition or subtraction) ***(M1)***

eg $\mathrm{AO} + \mathrm{OB},\ \mathrm{B} - \mathrm{A}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 9 \\ 6 \\ -3 \end{pmatrix}$ ***A1 N2***

[2 marks]

b. **METHOD 1**

valid approach using $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ ***(M1)***

eg $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix},\ \overrightarrow{\mathrm{CB}} = \begin{pmatrix} 6-x \\ 4-y \\ -1-z \end{pmatrix}$

correct working ***A1***

eg $\begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix} = \begin{pmatrix} 12-2x \\ 8-2y \\ -2-2z \end{pmatrix}$

all three equations ***A1***

eg $x+3 = 12-2x,\ y+2 = 8-2y,\ z-2 = -2-2z,$

$\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ ***AG N0***

METHOD 2

valid approach ***(M1)***

eg $\overrightarrow{OC} - \overrightarrow{OA} = 2\left(\overrightarrow{OB} - \overrightarrow{OC}\right)$

correct working ***A1***

eg $3\overrightarrow{OC} = 2\overrightarrow{OB} + \overrightarrow{OA}$

correct substitution of $\overrightarrow{OB}$ and $\overrightarrow{OA}$ ***A1***

eg $3\overrightarrow{OC} = 2\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix},\ 3\overrightarrow{OC} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$

$\overrightarrow{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ ***AG N0***

METHOD 3

valid approach ***(M1)***

eg $\overrightarrow{AC} = \frac{2}{3}\overrightarrow{AB}$, diagram, $\overrightarrow{CB} = \frac{1}{3}\overrightarrow{AB}$

correct working ***A1***

eg $\overrightarrow{AC} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix},\ \overrightarrow{CB} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

correct working involving $\overrightarrow{OC}$ ***A1***

eg $\overrightarrow{OC} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix},\ \begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

$\overrightarrow{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ ***AG N0***

[3 marks]

c. finding scalar product and magnitudes ***(A1)(A1)(A1)***

scalar product $= (9 \times 3) + (6 \times 2) + (-3 \times 0)\ (= 39)$

magnitudes $\sqrt{81 + 36 + 9}\ (= 11.22),\ \sqrt{9 + 4}\ (= 3.605)$

substitution into formula ***M1***

eg $\cos\theta = \frac{(9\times3)+12}{\sqrt{126}\times\sqrt{13}}$

$\theta = 0.270549\ (\text{accept } 15.50135^\circ)$

$\theta = 0.271\ (\text{accept } 15.5^\circ)$ ***A1 N4***

[5 marks]

d. (i) attempt to use a trig ratio ***M1***

eg $\sin\theta = \frac{DE}{CD},\ \left|\overrightarrow{CE}\right| = \left|\overrightarrow{CD}\right|\cos\theta$

attempt to express $\overrightarrow{CD}$ in terms of $\overrightarrow{OC}$ ***M1***

eg $\overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OD},\ OC + CD = OD$

correct working ***A1***

eg $\left|k\overrightarrow{\mathrm{OC}}-\overrightarrow{\mathrm{OC}}\right|\sin\theta$

$\left|\overrightarrow{\mathrm{DE}}\right|=(k-1)\left|\overrightarrow{\mathrm{OC}}\right|\sin\theta$ ***AG*** ***N0***

(ii) valid approach involving the segment DE ***(M1)***

eg recognizing $\left|\overrightarrow{\mathrm{DE}}\right|<3,\ \mathrm{DE}=3$

correct working (accept equation) ***(A1)***

eg $(k-1)(\sqrt{13})\sin 0.271<3,\ k-1=3.11324$

$1<k<4.11$ (accept $k<4.11$ but not $k=4.11$) ***A1*** ***N2***

[6 marks]

Examiners report

a. The majority of candidates had little difficulty with parts (a) and (c). The most common error in both these parts were unforced arithmetic errors and occasional misreads of the vectors.

b. In part (b), candidates who were successful used a variety of different approaches, and it was pleasing to see the vast majority of these being well reasoned, however, there were numerous unsuccessful responses including those who attempted to use the given vector to work backwards. A lack of appropriate vector notation often meant that ideas were not always clearly communicated.

c. The majority of candidates had little difficulty with parts (a) and (c). The most common error in both these parts were unforced arithmetic errors and occasional misreads of the vectors.

d. The majority of candidates struggled to make any progress in (d), with very few realizing that simple right-angled trigonometry could be used. Few were able to successfully express CD in terms of OC which was required to show the given result. Very few candidates attempted (d)(ii), with many unable to make the connection with results found in previous parts of the question.

Let $f(x)=x^2$ and $g(x)=3\ln(x+1)$, for $x>-1$.

a. Solve $f(x)=g(x)$. [3]

b. Find the area of the region enclosed by the graphs of f and g. [3]

Markscheme

a. valid approach ***(M1)***

eg sketch

0, 1.73843

$x=0,\ x=1.74$ (accept $(0,\ 0)$ and $(1.74,\ 3.02)$) ***A1A1*** ***N3***

[3 marks]

b. integrating and subtracting functions (in any order) ***(M1)***

eg $\int g - f$

correct substitution of their limits **or** function (accept missing $\mathrm{d}x$)

(A1)

eg $\int_0^{1.74} g - f,\ \int 3\ln(x+1) - x^2$

Note: Do not award ***A1*** if there is an error in the substitution.

1.30940

1.31 ***A1*** ***N3***

[3 marks]

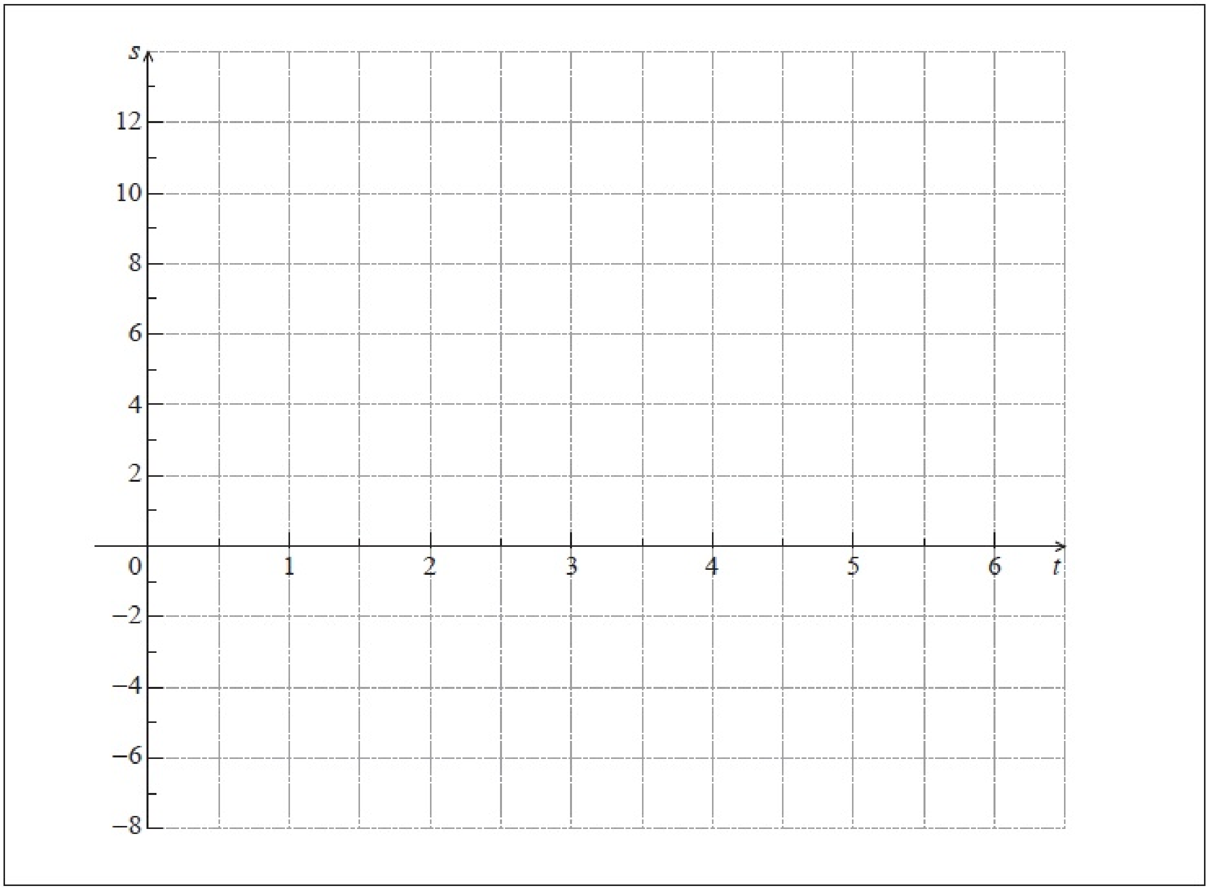
Examiners report

a. Candidates often did not make the connection between parts (a) and (b). The extraordinary number of failed analytical approaches in part (a) and correct use of the GDC to find the limits in part (b) suggests that candidates are equating the command term "solve" to mean use an algebraic approach to solve equations or inequalities, instead of their GDC. Many candidates appeared to interpret part (a) as something they should do by hand and often did not recognize that their answer to part (a) were the limits in part (b). Quite a few candidates failed to interpret a GDC solution of $x = 5 \times 10^{-14}$ correctly as $x = 0$ and others found the solution $x = 1.74$ as the only solution, ignoring the second intersection point until part (b).

b. Candidates often did not make the connection between parts (a) and (b). The extraordinary number of failed analytical approaches in part (a) and correct use of the GDC to find the limits in part (b) suggests that candidates are equating the command term "solve" to mean use an algebraic approach to solve equations or inequalities, instead of their GDC. Many candidates appeared to interpret part (a) as something they should do by hand and often did not recognize that their answer to part (a) were the limits in part (b). Quite a few candidates failed to interpret a GDC solution of $x = 5 \times 10^{-14}$ correctly as $x = 0$ and others found the solution $x = 1.74$ as the only solution, ignoring the second intersection point until part (b).

A particle's displacement, in metres, is given by $s(t) = 2t\cos t$, for $0 \le t \le 6$, where t is the time in seconds.

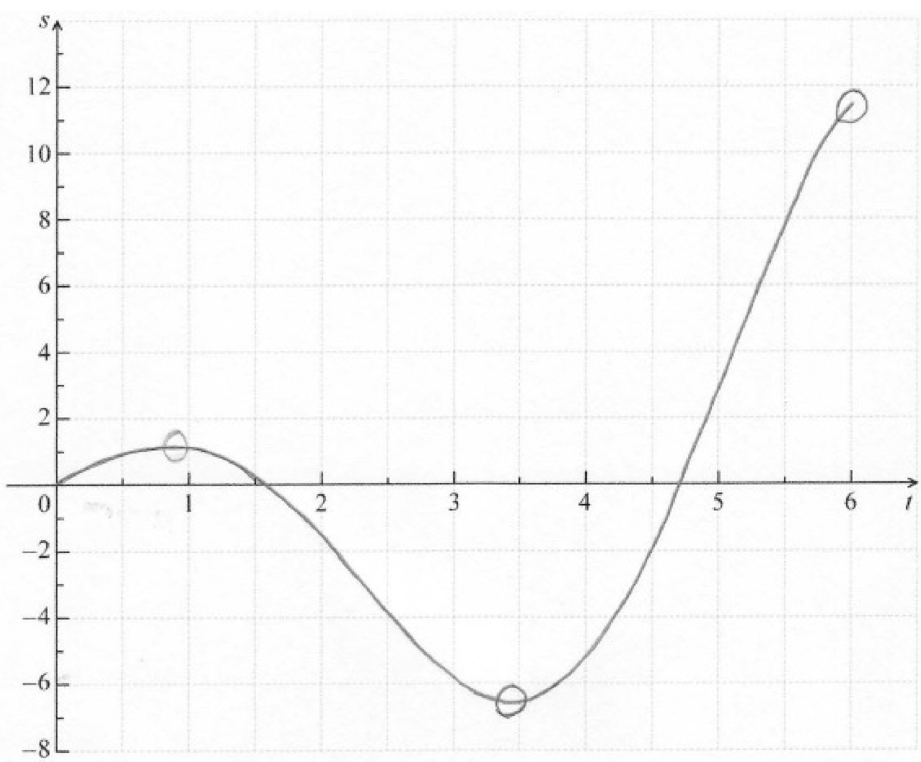
a. On the grid below, sketch the graph of s . [4]



b. Find the maximum velocity of the particle. [3]

Markscheme

a.



A1A1A1A1 ***N4***

Note: Award ***A1*** for approximately correct shape (do not accept line segments).

Only if this ***A1*** is awarded, award the following:

A1 for maximum and minimum within circles,

A1 for x-intercepts between 1 and 2 **and** between 4 and 5,

A1 for left endpoint at $(0, 0)$ and right endpoint within circle.

[4 marks]

b. appropriate approach ***(M1)***

e.g. recognizing that $v = s'$, finding derivative, $a = s''$

valid method to find maximum ***(M1)***

e.g. sketch of v , $v'(t) = 0$, $t = 5.08698\dots$

$v = 10.20025\dots$

$v = 10.2$ $[10.2, 10.3]$ ***A1*** ***N2***

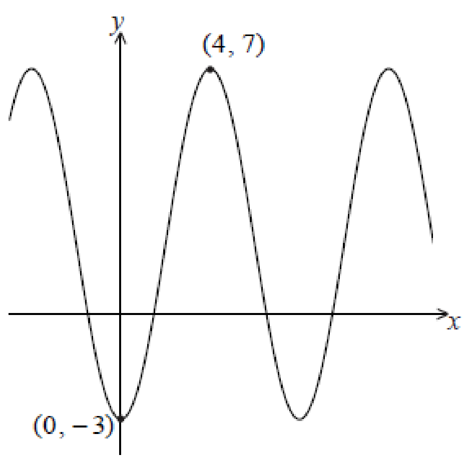
[3 marks]

Examiners report

a. Most candidates sketched an approximately correct shape for the displacement of a particle in the given domain, but many lost marks for carelessness in graphing the local extrema or the right endpoint.

b. In part (b), most candidates knew to differentiate displacement to find velocity, but few knew how to then find the maximum. Occasionally, a candidate would give the time value of the maximum. Others attempted to incorrectly set the first derivative equal to zero and solve analytically rather than take the maximum value from the graph of the velocity function.

The graph of $y = p\cos qx + r$, for $-5 \le x \le 14$, is shown below.



There is a minimum point at (0, −3) and a maximum point at (4, 7) .

a(i), (ii) and (iii). Find the value of [6]

(i) p ;

(ii) q ;

(iii) r.

b. The equation $y = k$ has exactly **two** solutions. Write down the value of k. [1]

Markscheme

a(i), (ii) and (iii). evidence of finding the amplitude *(M1)*

e.g. $\frac{7+3}{2}$, amplitude $= 5$

$p = -5$ ***A1 N2***

(ii) period $= 8$ *(A1)*

$q = 0.785 \left(= \frac{2\pi}{8} = \frac{\pi}{4}\right)$ ***A1 N2***

(iii) $r = \frac{7-3}{2}$ *(A1)*

$r = 2$ ***A1 N2***

[6 marks]

b. $k = -3$ (accept $y = -3$) ***A1 N1***

[1 mark]

Examiners report

a(i), (ii) and (iii). Many candidates did not recognize that the value of p was negative. The value of q was often interpreted incorrectly as the period but most candidates could find the value of r, the vertical translation.

b. In part (b), candidates either could not find a solution or found too many.

Let $f(x) = 2x + 3$ and $g(x) = x^3$.

a. Find $(f \circ g)(x)$. [2]

b. Solve the equation $(f \circ g)(x) = 0$. [3]

Markscheme

a. attempt to form composite (in any order) ***(M1)***

eg $f(x^3),\ (2x+3)^3$

$(f \circ g)(x) = 2x^3 + 3$ ***A1 N2***

[2 marks]

b. evidence of appropriate approach ***(M1)***

eg $2x^3 = -3$, sketch

correct working ***(A1)***

eg $x^3 = \frac{-3}{2}$, sketch

-1.14471

$x = \sqrt[3]{\frac{-3}{2}}$ $\text{(exact)}, \ -1.14 \ [-1.15, \ -1.14]$ ***A1*** ***N3***

[3 marks]

Total [5 marks]

Examiners report

a. Generally well done, though there were some careless errors with the substitution into f in part (ai) and rearranging the equation in part (b). Although candidates understood that they were supposed to solve the equation $2x^3 + 3 = 0$, many wrote $2x^3 = 3$ or $x = \sqrt{\frac{3}{2}}$. The majority of the candidates chose an algebraic method instead of using their GDC.

b. Generally well done, though there were some careless errors with the substitution into f in part (ai) and rearranging the equation in part (b). Although candidates understood that they were supposed to solve the equation $2x^3 + 3 = 0$, many wrote $2x^3 = 3$ or $x = \sqrt{\frac{3}{2}}$. The majority of the candidates chose an algebraic method instead of using their GDC.

Let $f(x) = -x^4 + 2x^3 - 1$, for $0 \le x \le 2$.

a. Sketch the graph of f on the following grid. [3]

b. Solve $f(x) = 0$. [2]

c. The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. [3]

Find the volume of the solid formed.

Markscheme

a. ***A1A1A1*** ***N3***

Note: Award ***A1*** for both endpoints in circles,

A1 for approximately correct shape (concave up to concave down).

Only if this ***A1*** for shape is awarded, award ***A1*** for maximum point in circle.

b. $x = 1$ $\quad x = 1.83928$

$x = 1 \ \text{(exact)}$ $\quad x = 1.84 \ [1.83, \ 1.84]$ ***A1A1*** ***N2***

[2 marks]

c. attempt to substitute either (***FT***) limits or function into formula with f^2 ***(M1)*** *eg* $V = \pi \int_1^{1.84} f^2, \ \int \left(-x^4 + 2x^3 - 1\right)^2 \mathrm{d}x$

0.636581

$V = 0.637 \ [0.636, \ 0.637]$ ***A2*** ***N3***

[3 marks]

Total [8 marks]

Examiners report

a. Despite being a straightforward question, and although most candidates had a roughly correct shape for their graph, their sketches were either out of scale or missed one of the endpoints. In part (b), a few did not give both answers despite going on to use 1.84 in part (c).

   Part (c) proved difficult for most candidates, as only a small number could write the correct expression for the volume: some included the correct limits but did not square the function, whilst others squared the function but did not write the correct limits in the integral. Many did not find a volume, or found an incorrect volume. The latter included finding the integral from 0 to 2, or dividing the region into three parts, showing a lack of understanding of “enclosed”.

b. Despite being a straightforward question, and although most candidates had a roughly correct shape for their graph, their sketches were either out of scale or missed one of the endpoints. In part (b), a few did not give both answers despite going on to use 1.84 in part (c).

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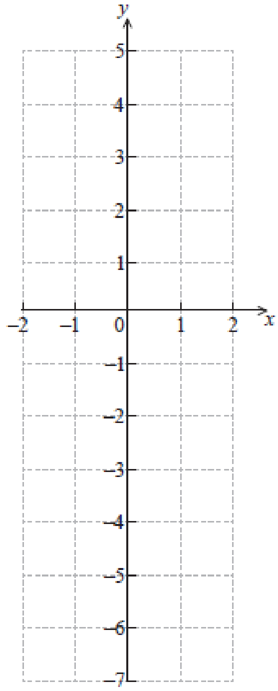
Let $f(x) = \cos(\mathrm{e}^x)$, for $-2 \le x \le 2$.

a. Find $f'(x)$. [2]

b. On the grid below, sketch the graph of $f'(x)$. [4]

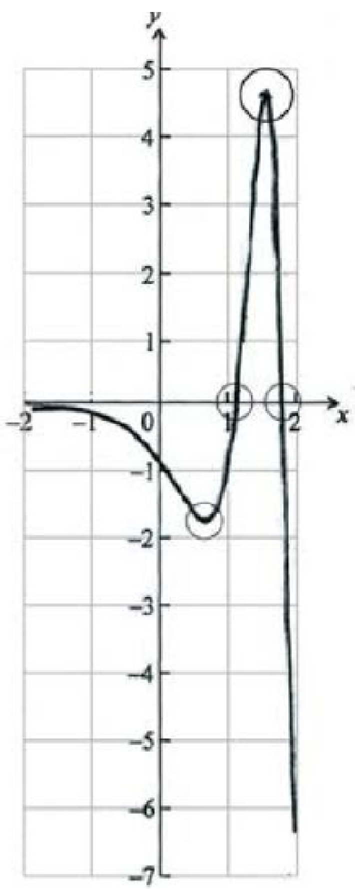


Markscheme

a. $f'(x) = -e^x \sin(e^x)$ ***A1A1*** ***N2***

[2 marks]

b.



A1A1A1A1 ***N4***

Note: Award ***A1*** for shape that must have the correct domain (from -2 to $+2$) and correct range (from -6 to 4), ***A1*** for minimum in circle, ***A1*** for maximum in circle and ***A1*** for intercepts in circles.

[4 marks]

Examiners report

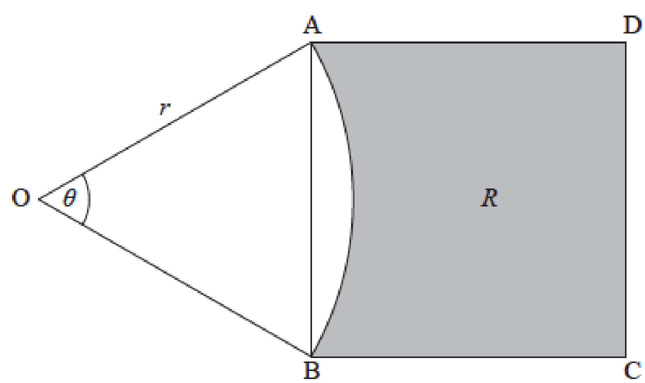
a. Many students failed in applying the chain rule to find the correct derivative, and some inappropriately used the product rule. However, many of those obtained full follow through marks in part (b) for the sketch of the function they found in part (a).

b. Many students failed in applying the chain rule to find the correct derivative, and some inappropriately used the product rule. However, many of those obtained full follow through marks in part (b) for the sketch of the function they found in part (a).

Most candidates sketched an approximately correct shape in the given domain, though there were some that did not realize they had to set their GDC to radians, producing a meaningless sketch.

It is very important to stress to students that although they are asked to produce a sketch, it is still necessary to show its key features such as domain and range, stationary points and intercepts.

The following diagram shows a square $ABCD$, and a sector OAB of a circle centre O, radius r. Part of the square is shaded and labelled R.



$\mathrm{A\hat{O}B} = \theta$, where $0.5 \leq \theta < \pi$.

a. Show that the area of the square $ABCD$ is $2r^2(1 - \cos\theta)$. [4]

b. When $\theta = \alpha$, the area of the square $ABCD$ is equal to the area of the sector OAB. [4]

(i) Write down the area of the sector when $\theta = \alpha$.

(ii) Hence find α.

c. When $\theta = \beta$, the area of R is more than twice the area of the sector. [8]

Find all possible values of β.

Markscheme

a. area of $\mathrm{ABCD} = \mathrm{AB}^2$ (seen anywhere) ***(A1)***

choose cosine rule to find a side of the square ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos\theta$

correct substitution (for triangle AOB) ***A1***

eg $r^2 + r^2 - 2 \times r \times r\cos\theta,\ \mathrm{OA}^2 + \mathrm{OB}^2 - 2 \times \mathrm{OA} \times \mathrm{OB}\cos\theta$

correct working for AB^2 ***A1***

eg $2r^2 - 2r^2 \cos\theta$

$\text{area} = 2r^2(1 - \cos\theta)$ ***AG*** ***N0***

Note: Award no marks if the only working is $2r^2 - 2r^2 \cos\theta$.

[4 marks]

b. (i) $\frac{1}{2}\alpha r^2 \quad \left(\text{accept } 2r^2(1-\cos\alpha)\right)$ ***A1*** ***N1***

(ii) correct equation in one variable ***(A1)***

eg $2(1-\cos\alpha) = \frac{1}{2}\alpha$

$\alpha = 0.511024$

$\alpha = 0.511 \quad (\text{accept } \theta = 0.511)$ ***A2*** ***N2***

Note: Award ***A1*** for $\alpha = 0.511$ and additional answers.

[4 marks]

c. **Note:** In this part, accept θ instead of β, and the use of equations instead of inequalities in the working.

attempt to find R ***(M1)***

eg subtraction of areas, $\text{square} - \text{segment}$

correct expression for segment area ***(A1)***

eg $\frac{1}{2}\beta r^2 - \frac{1}{2}r^2 \sin\beta$

correct expression for R ***(A1)***

eg $2r^2(1-\cos\beta) - \left(\frac{1}{2}\beta r^2 - \frac{1}{2}r^2\sin\beta\right)$

correct inequality ***(A1)***

eg $2r^2(1-\cos\beta) - \left(\frac{1}{2}\beta r^2 - \frac{1}{2}r^2\sin\beta\right) > 2\left(\frac{1}{2}\beta r^2\right)$

correct inequality in terms of angle only ***A1***

eg $2(1-\cos\beta) - \left(\frac{1}{2}\beta - \frac{1}{2}\sin\beta\right) > \beta$

attempt to solve their inequality, must represent $R >$ twice sector ***(M1)***

eg sketch, one correct value

Note: Do not award the second ***(M1)*** unless the first ***(M1)*** for attempting to find R has been awarded.

both correct values 1.30573 and 2.67369 ***(A1)***

correct inequality $1.31 < \beta < 2.67$ ***A1*** ***N3***

[8 marks]

Total [16 marks]

Examiners report

a. Those who attempted part (a) could in general show what was required by using the cosine rule. On rare occasions some more complicated approaches were seen using half of angle theta. In some cases, candidates did not show all the necessary steps and lost marks for not completely showing the given result.

b. A number of candidates correctly answered part (bi) and created a correct equation in (bii), but did not solve the equation correctly, usually attempting an analytic method where the GDC would do. For many a major problem was to realize the need to reduce the equation to one variable before attempting to solve it. Occasionally, an answer would be written that was outside the given domain.

c. When part (c) was attempted, many candidates did not recognize that the area in question requires the subtraction of a segment area, and often set the square area greater than twice the sector. Many candidates made mistakes when trying to eliminate brackets or just did not use them. Of those who created a correct inequality, few reached a fully correct conclusion.

Let $f(x) = kx^2 + kx$ and $g(x) = x - 0.8$. The graphs of f and g intersect at two distinct points.

Find the possible values of k.

Markscheme

attempt to set up equation ***(M1)***

eg $f = g,\ kx^2 + kx = x - 0.8$

rearranging **their** equation to equal zero ***M1***

eg $kx^2 + kx - x + 0.8 = 0,\ kx^2 + x(k-1) + 0.8 = 0$

evidence of discriminant (if seen explicitly, not just in quadratic formula) ***(M1)***

eg $b^2 - 4ac,\ \Delta = (k-1)^2 - 4k \times 0.8,\ D = 0$

correct discriminant ***(A1)***

eg $(k-1)^2 - 4k \times 0.8,\ k^2 - 5.2k + 1$

evidence of correct discriminant greater than zero ***R1***

eg $k^2 - 5.2k + 1 > 0,\ (k-1)^2 - 4k \times 0.8 > 0$, correct answer

both correct values ***(A1)***

eg $0.2,\ 5$

correct answer ***A2*** ***N3***

eg $k < 0.2,\ k \neq 0,\ k > 5$

[8 marks]

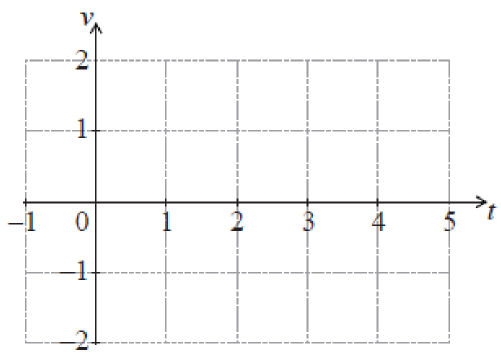
Examiners report

Many candidates knew to set the equations equal, and then some knew to manipulate the equation such that it is equal to zero. Those who recognized the discriminant in this equation earned further marks, although few set a correct discriminant greater than zero. Even in such cases, finding both inequalities proved elusive for most.

An alternative method was to graph each function and find where the line intersects the parabola in exactly one and in two places. Few could carry this approach to adequate completion, often neglecting a second inequality for k.

The velocity of a particle in ms^{-1} is given by $v = \mathrm{e}^{\sin t} - 1$, for $0 \le t \le 5$.

a. On the grid below, sketch the graph of v . [3]

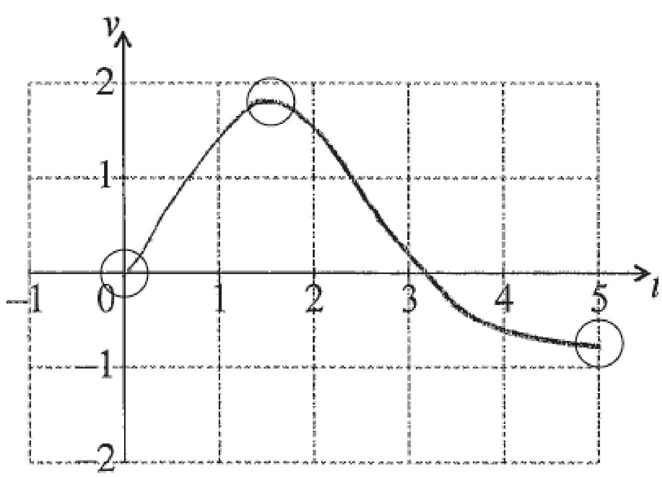


b.i. Find the total distance travelled by the particle in the first five seconds. [1]

b.ii. Write down the positive t-intercept. [4]

Markscheme

a.



A1A1A1 *N3*

Note: Award ***A1*** for approximately correct shape crossing x-axis with $3 < x < 3.5$.

Only if this ***A1*** is awarded, award the following:

A1 for maximum in circle, ***A1*** for endpoints in circle.

[3 marks]

b.i. $t = \pi$ (exact), 3.14 ***A1*** ***N1***

[1 mark]

b.ii. recognizing distance is area under velocity curve ***(M1)*** *eg* $s = \int v$, shading on diagram, attempt to integrate

valid approach to find the total area ***(M1)***

eg area A + area B , $\int v\mathrm{d}t - \int v\mathrm{d}t$, $\int_0^{3.14} v\mathrm{d}t + \int_{3.14}^{5} v\mathrm{d}t$, $\int |v|$

correct working with integration and limits (accept $\mathrm{d}x$ or missing $\mathrm{d}t$) ***(A1)***

eg $\int_0^{3.14} v\mathrm{d}t + \int_5^{3.14} v\mathrm{d}t$, $3.067\ldots + 0.878\ldots$, $\int_0^5 |\mathrm{e}^{\sin t} - 1|$

distance $= 3.95$ (m) ***A1*** ***N3***

[4 marks]

Examiners report

a. There was a minor error on this question, where the units for velocity were given as ms^{-2} rather than ms^{-1} . Examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were considered at the grade award meeting.

Candidates continue to produce sloppy graphs resulting in loss of marks. Although the shape was often correctly drawn, students were careless when considering the domain and other key features such as the root and the location of the maximum point.

b.i. The fact that most candidates with poorly drawn graphs correctly found the root in (b)(i), clearly emphasized the disconnect between geometric and algebraic approaches to problems.

b.ii. In (b)(ii), most appreciated that the definite integral would give the distance travelled but few could write a valid expression and normally just integrated from $t = 0$ to $t = 5$ without considering the part of the graph below the t-axis. Again, analytic approaches to evaluating their integral predominated over simpler GDC approaches and some candidates had their calculator set in degree mode rather than radian mode.

Consider a geometric sequence where the first term is 768 and the second term is 576.

Find the least value of n such that the nth term of the sequence is less than 7.

Markscheme

attempt to find r ***(M1)***

eg $\frac{576}{768}, \frac{768}{576}, 0.75$

correct expression for u_n ***(A1)***

eg $768(0.75)^{n-1}$

EITHER (solving inequality)

valid approach (accept equation) ***(M1)***

eg $u_n < 7$

valid approach to find n ***M1***

eg $768(0.75)^{n-1} = 7,\ n - 1 > \log_{0.75}\left(\frac{7}{768}\right)$, sketch

correct value

eg $n = 17.3301$ ***(A1)***

$n = 18$ (must be an integer) ***A1*** ***N2***

OR (table of values)

valid approach ***(M1)***

eg $u_n > 7$, one correct crossover value

both crossover values, $u_{17} = 7.69735$ **and** $u_{18} = 5.77301$ ***A2***

$n = 18$ (must be an integer) ***A1*** ***N2***

OR (sketch of functions)

valid approach ***M1***

eg sketch of appropriate functions

valid approach ***(M1)***

eg finding intersections or roots (depending on function sketched)

correct value

eg $n = 17.3301$ ***(A1)***

$n = 18$ (must be an integer) ***A1*** ***N2***

[6 marks]

Examiners report

[N/A]

A rock falls off the top of a cliff. Let h be its height above ground in metres, after t seconds.

The table below gives values of h and t .

t (seconds)	1	2	3	4	5
h (metres)	105	98	84	60	26

a(i), (ii) and (iii). Jane thinks that the function $f(t) = -0.25t^3 - 2.32t^2 + 1.93t + 106$ is a suitable model for the data. Use Jane's model to [5]

(i) write down the height of the cliff;

(ii) find the height of the rock after 4.5 seconds;

(iii) find after how many seconds the height of the rock is 30 m.

b. Kevin thinks that the function $g(t) = -5.2t^2 + 9.5t + 100$ is a better model for the data. Use Kevin's model to find when the rock hits the ground. [3]

c(i) and (ii). (i) On graph paper, using a scale of 1 cm to 1 second, and 1 cm to 10 m, plot the data given in the table. [6]

(ii) By comparing the graphs of f and g with the plotted data, explain which function is a better model for the height of the falling rock.

Markscheme

a(i), (ii) and (iii). (i) 106 m ***A1*** ***N1***

(ii) substitute $t = 4.5$ ***M1***

$h = 44.9$ m ***A1*** ***N2***

(iii) set up suitable equation ***M1***

e.g. $f(t) = 30$

$t = 4.91$ ***A1*** ***N1***

[5 marks]

b. recognizing that height is 0 ***A1***

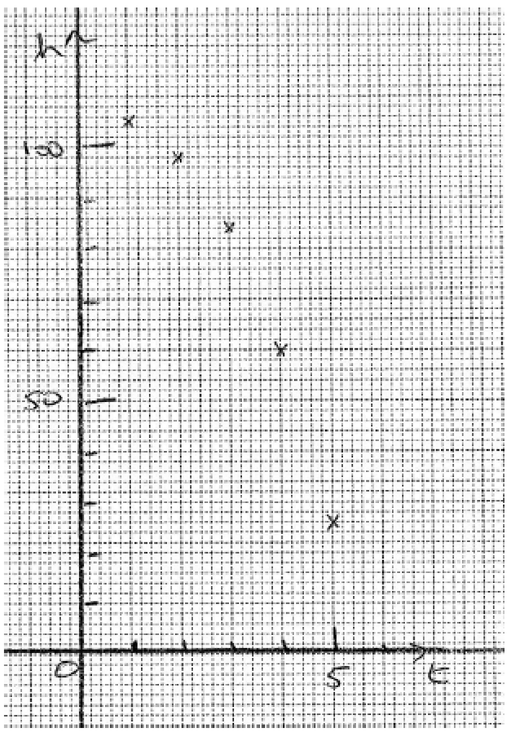
set up suitable equation ***M1***

e.g. $g(t) = 0$

$t = 5.39$ secs ***A1*** ***N2***

[3 marks]

c(i) and (ii).



A1A2 ***N3***

Note: Award ***A1*** for correct scales on axes, ***A2*** for 5 correct points, ***A1*** for 3 or 4 correct points.

(ii) Jane’s function, with **2** valid reasons ***A1R1R1*** ***N3***

e.g. Jane’s passes very close to all the points, Kevin’s has the rock clearly going up initially – not possible if rock falls

Note: Although Jane’s also goes up initially, it only goes up very slightly, and so is the better model.

[6 marks]

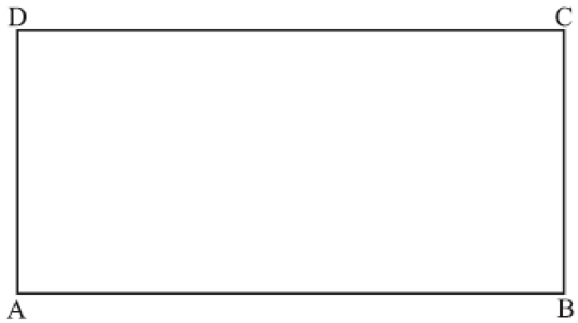
Examiners report

a(i), (ii) and (iii).
[N/A]

b.
[N/A]

c(i) and (ii).
[N/A]

A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m^2, as shown below.



The fencing used for side AB costs \$11 per metre. The fencing for the other three sides costs \$3 per metre. The farmer creates an enclosure so that the cost is a minimum. Find this minimum cost.

Markscheme

METHOD 1

correct expression for **second** side, using area = 525 ***(A1)***

e.g. let $\mathrm{AB} = x$, $\mathrm{AD} = \frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side ***(M1)***

e.g. $3(\mathrm{AD} + \mathrm{BC} + \mathrm{CD}) + 11\mathrm{AB}$

correct expression for cost ***A2***

e.g. $\frac{525}{x} \times 3 + \frac{525}{x} \times 3 + 11x + 3x$, $\frac{525}{\mathrm{AB}} \times 3 + \frac{525}{\mathrm{AB}} \times 3 + 11\mathrm{AB} + 3\mathrm{AB}$, $\frac{3150}{x} + 14x$

EITHER

sketch of cost function ***(M1)***

identifying minimum point ***(A1)***

e.g. marking point on graph, $x = 15$

minimum cost is 420 (dollars) ***A1*** ***N4***

OR

correct derivative (may be seen in equation below) ***(A1)***

e.g. $C'(x) = \frac{-1575}{x^2} + \frac{-1575}{x^2} + 14$

setting their derivative equal to 0 (seen anywhere) ***(M1)***

e.g. $\frac{-3150}{x^2} + 14 = 0$

minimum cost is 420 (dollars) ***A1*** ***N4***

METHOD 2

correct expression for **second** side, using area = 525 ***(A1)***

e.g. let $\mathrm{AD} = x$, $\mathrm{AB} = \frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side ***(M1)***

e.g. $3(\mathrm{AD} + \mathrm{BC} + \mathrm{CD}) + 11\mathrm{AB}$

correct expression for cost ***A2***

e.g. $3\left(x + x + \frac{525}{x}\right) + \frac{525}{x} \times 11$, $3\left(\mathrm{AD} + \mathrm{AD} + \frac{525}{\mathrm{AD}}\right) + \frac{525}{\mathrm{AD}} \times 11$, $6x + \frac{7350}{x}$

EITHER

sketch of cost function ***(M1)***

identifying minimum point ***(A1)***

e.g. marking point on graph, $x = 35$

minimum cost is 420 (dollars) ***A1 N4***

OR

correct derivative (may be seen in equation below) ***(A1)***

e.g. $C'(x) = 6 - \frac{7350}{x^2}$

setting their derivative equal to 0 (seen anywhere) ***(M1)***

e.g. $6 - \frac{7350}{x^2} = 0$

minimum cost is 420 (dollars) ***A1 N4***

[7 marks]

Examiners report

Although this question was a rather straight-forward optimisation question, the lack of structure caused many candidates difficulty. Some were able to calculate cost values but were unable to create an algebraic cost function. Those who were able to create a cost function in two variables often could not use the area relationship to obtain a function in a single variable and so could make no further progress. Of those few who created a correct cost function, most set the derivative to zero to find that the minimum cost occurred at $x = 15$, leading to $420. Although this is a correct approach earning full marks, candidates seem not to recognise that the result can be obtained from the GDC, without the use of calculus.

The first two terms of a geometric sequence u_n are $u_1 = 4$ and $u_2 = 4.2$.

a. (i) Find the common ratio. [5]

(ii) Hence or otherwise, find u_5.

b. Another sequence v_n is defined by $v_n = an^k$, where $a,\ k \in \mathbb{R}$, and $n \in \mathbb{Z}^+$, such that $v_1 = 0.05$ and $v_2 = 0.25$. [5]

(i) Find the value of a.

(ii) Find the value of k.

c. Find the smallest value of n for which $v_n > u_n$. [5]

Markscheme

a. (i) valid approach ***(M1)***

eg $r = \frac{u_2}{u_1},\ \frac{4}{4.2}$

$r = 1.05 \quad (\text{exact})$ ***A1 N2***

(ii) attempt to substitute into formula, with **their** r ***(M1)***

eg $4 \times 1.05^n,\ 4 \times 1.05 \times 1.05 \ldots$

correct substitution ***(A1)***

eg $4 \times 1.05^4,\ 4 \times 1.05 \times 1.05 \times 1.05 \times 1.05$

$u_5 = 4.862025 \text{ (exact)}, 4.86\ [4.86,\ 4.87]$ ***A1*** ***N2***

[5 marks]

b. (i) attempt to substitute $n = 1$ ***(M1)***

eg $0.05 = a \times 1^k$

$a = 0.05$ ***A1*** ***N2***

(ii) correct substitution of $n = 2$ into v_2 ***A1***

eg $0.25 = a \times 2^k$

correct work ***(A1)***

eg finding intersection point, $k = \log_2\left(\frac{0.25}{0.05}\right),\ \frac{\log 5}{\log 2}$

2.32192

$k = \log_2 5 \quad \text{(exact)}, 2.32\ [2.32,\ 2.33]$ ***A1*** ***N2***

[5 marks]

c. correct expression for u_n ***(A1)***

eg $4 \times 1.05^{n-1}$

EITHER

correct substitution into inequality (accept equation) ***(A1)***

eg $0.05 \times n^k > 4 \times 1.05^{n-1}$

valid approach to solve inequality (accept equation) ***(M1)***

eg finding point of intersection, $n = 7.57994\ (7.59508 \text{ from } 2.32)$

$n = 8$ (must be an integer) ***A1*** ***N2***

OR

table of values

when $n = 7,\ u_7 = 5.3604,\ v_7 = 4.5836$ ***A1***

when $n = 8,\ u_8 = 5.6284,\ v_8 = 6.2496$ ***A1***

$n = 8$ (must be an integer) ***A1*** ***N2***

[4 marks]

Total [14 marks]

Examiners report

a. Most candidates answered part (a) correctly.

b. A surprising number assumed the second sequence to be geometric as well, and thus part (b) was confusing for many. It was quite common that students did not clearly show which work was relevant to part (i) and which to part (ii), thus often losing marks.

c. Few students successfully completed part (c) as tried to solve algebraically instead of graphically. Those who used the table of values did not always show two sets of values and consequently lost marks.

Let $f(x) = e^{2\sin\left(\frac{\pi x}{2}\right)}$, for $x > 0$.

The kth maximum point on the graph of f has x-coordinate x_k where $k \in \mathbb{Z}^+$.

a. Given that $x_{k+1} = x_k + a$, find a. [4]

b. Hence find the value of n such that $\sum_{k=1}^{n} x_k = 861$. [4]

Markscheme

a. valid approach to find maxima ***(M1)***

eg one correct value of x_k, sketch of f

any two correct consecutive values of x_k ***(A1)(A1)***

eg $x_1 = 1$, $x_2 = 5$

$a = 4$ ***A1 N3***

[4 marks]

b. recognizing the sequence x_1, x_2, x_3, ..., x_n is arithmetic ***(M1)***

eg $d = 4$

correct expression for sum ***(A1)***

eg $\frac{n}{2}(2(1) + 4(n-1))$

valid attempt to solve for n ***(M1)***

eg graph, $2n^2 - n - 861 = 0$

$n = 21$ ***A1 N2***

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

a. Find an expression for $h(x)$. [3]

b. Write down the period of h. [1]

c. Write down the range of h. [2]

Markscheme

a. attempt to form any composition (even if order is reversed) ***(M1)*** correct composition $h(x) = g\left(\frac{3x}{2} + 1\right)$ ***(A1)***

$h(x) = 4\cos\left(\frac{\frac{3x}{2}+1}{3}\right) - 1 \quad \left(4\cos\left(\frac{1}{2}x + \frac{1}{3}\right) - 1, 4\cos\left(\frac{3x+2}{6}\right) - 1\right)$ ***A1 N3***

[3 marks]

b. period is $4\pi(12.6)$ ***A1 N1***

[1 mark]

c. range is $-5 \leq h(x) \leq 3 \ ([-5, 3])$ ***A1A1 N2***

[2 marks]

Examiners report

a. The majority of candidates handled the composition of the two given functions well. However, a large number of candidates had difficulties simplifying the result correctly. The period and range of the resulting trig function was not handled well. If candidates knew the definition of "range", they often did not express it correctly. Many candidates correctly used their GDCs to find the period and range, but this approach was not the most efficient.

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Let $f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right)$, for $-4 \leqslant x \leqslant 4$.

a. Sketch the graph of f. [3]

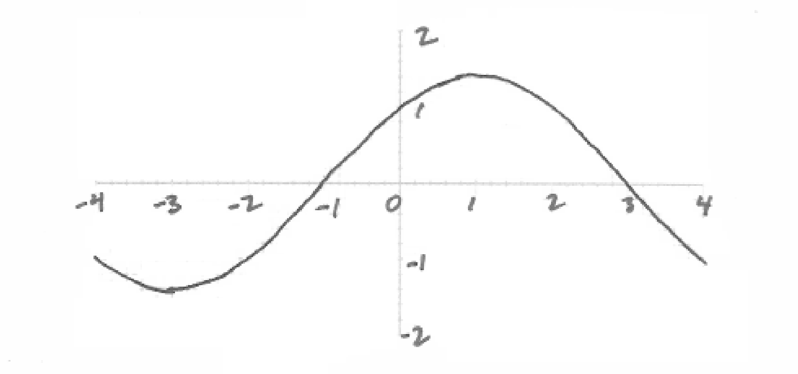
b. Find the values of x where the function is decreasing. [5]

c(i). The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x + c)\right)$, where $a \in \mathbb{R}$, and $0 \leqslant c \leqslant 2$. Find the value of a; [3]

c(ii). The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x + c)\right)$, where $a \in \mathbb{R}$, and $0 \leqslant c \leqslant 2$. Find the value of c. [4]

Markscheme

a.



A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct sinusoidal shape.

Only if this ***A1*** is awarded, award the following:

A1 for correct domain,

A1 for approximately correct range.

[3 marks]

b. recognizes decreasing to the left of minimum or right of maximum,

eg $f'(x) < 0$ ***(R1)***

x-values of minimum and maximum (may be seen on sketch in part (a)) ***(A1)(A1)***

eg $x = -3,\ (1, 1.4)$

two correct intervals ***A1A1*** ***N5***

eg $-4 < x < -3,\ 1 \leqslant x \leqslant 4;\ x < -3,\ x \geqslant 1$

[5 marks]

c(i). recognizes that a is found from amplitude of wave ***(R1)***

y-value of minimum or maximum ***(A1)***

eg $(-3, -1.41), (1, 1.41)$

$a = 1.41421$

$a = \sqrt{2},\ \text{(exact)}, 1.41,$ ***A1*** ***N3***

[3 marks]

c(ii). **METHOD 1**

recognize that shift for sine is found at x-intercept ***(R1)***

attempt to find x-intercept ***(M1)***

eg $\cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right) = 0,\ x = 3 + 4k,\ k \in \mathbb{Z}$

$x = -1$ ***(A1)***

$c = 1$ ***A1*** ***N4***

METHOD 2

attempt to use a coordinate to make an equation ***(R1)***

eg $\sqrt{2}\sin\left(\frac{\pi}{4}c\right) = 1,\ \sqrt{2}\sin\left(\frac{\pi}{4}(3 - c)\right) = 0$

attempt to solve resulting equation ***(M1)***

eg sketch, $x = 3 + 4k,\ k \in \mathbb{Z}$

$x = -1$ ***(A1)***

$c = 1$ ***A1*** ***N4***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c(i). [N/A]
c(ii). [N/A]

A particle moves in a straight line. Its velocity $v \text{ m s}^{-1}$ after t seconds is given by

$$v = 6t - 6, \text{ for } 0 \leqslant t \leqslant 2.$$

After p seconds, the particle is 2 m from its initial position. Find the possible values of p.

Markscheme

correct approach ***(A1)***

eg $s = \int v, \int_0^p 6t - 6\mathrm{d}t$

correct integration ***(A1)***

eg $\int 6t - 6\mathrm{d}t = 3t^2 - 6t + C, \left[3t^2 - 6t\right]_0^p$

recognizing that there are two possibilities ***(M1)***

eg 2 correct answers, $s = \pm 2, c \pm 2$

two correct equations in p ***A1A1***

eg $3p^2 - 6p = 2, 3p^2 - 6p = -2$

0.42265, 1.57735

$p = 0.423 \text{ or } p = 1.58$ ***A1A1*** ***N3***

[7 marks]

Examiners report

Most candidates realized that they needed to calculate the integral of the velocity, and did it correctly. However, only a few realized that there were two possible positions for the particle, as it could move in two directions. In general, the only equation candidates wrote was $3p^2 - 6p = 2$, that gave solutions outside the given domain. Candidates failed to differentiate between displacement and distance travelled.

Consider the expansion of $(x + 2)^{11}$.

a. Write down the number of terms in this expansion. [1]

b. Find the term containing x^2 . [4]

Markscheme

a. 12 terms ***A1*** ***N1***

[1 mark]

b. evidence of binomial expansion ***(M1)***

e.g. $\binom{n}{r} a^{n-r}b^r$, an attempt to expand, Pascal's triangle

evidence of choosing correct term ***(A1)***

e.g. 10th term , $r = 9$, $\binom{11}{9}$, $(x)^2(2)^9$

correct working ***A1***

e.g. $\binom{11}{9} (x)^2(2)^9$, 55×2^9

$28160x^2$ ***A1*** ***N2***

[4 marks]

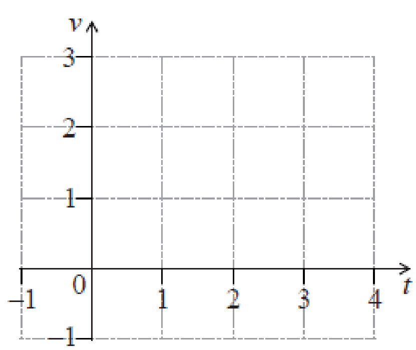
Examiners report

a. Most candidates attempted this question, and many made good progress. A number of candidates spent time writing out Pascal's triangle in full. Common errors included 11 for part (a) and not writing out the simplified form of the term for part (b). Another common error was adding instead of multiplying the parts of the term in part (b).

b. Most candidates attempted this question, and many made good progress. A number of candidates spent time writing out Pascal's triangle in full. Common errors included 11 for part (a) and not writing out the simplified form of the term for part (b). Another common error was adding instead of multiplying the parts of the term in part (b).

A particle moves along a straight line such that its velocity, v ms^{-1}, is given by $v(t) = 10te^{-1.7t}$, for $t \geqslant 0$.

a. On the grid below, sketch the graph of v, for $0 \leqslant t \leqslant 4$. [3]

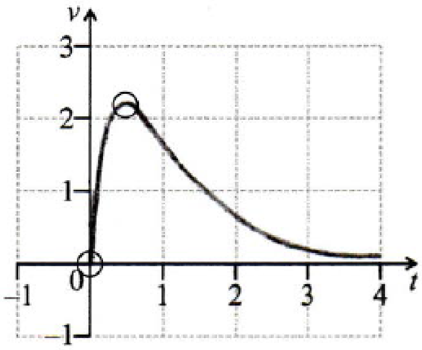


b. Find the distance travelled by the particle in the first three seconds. [2]

c. Find the velocity of the particle when its acceleration is zero. [3]

Markscheme

a.



A1A2 N3

Notes: Award ***A1*** for approximately correct domain $0 \leqslant t \leqslant 4$.
The shape must be approximately correct, with maximum skewed left. **Only** if the shape is approximately correct, award ***A2*** for all the following approximately correct features, in circle of tolerance where drawn (accept seeing correct coordinates for the maximum, even if point outside circle):
Maximum point, passes through origin, asymptotic to t-axis (but must not touch the axis).
If only two of these features are correct, award ***A1***.

[3 marks]

b. valid approach (including 0 and 3) ***(M1)***

eg $\int_0^3 10te^{-1.7t}\mathrm{d}t,\ \int_0^3 f(x)$, area from 0 to 3 (may be shaded in diagram)
distance $= 3.33$ (m) ***A1 N2***

[2 marks]

c. recognizing acceleration is derivative of velocity ***(R1)***

eg $a = \frac{\mathrm{d}v}{\mathrm{d}t}$, attempt to find $\frac{\mathrm{d}v}{\mathrm{d}t}$, reference to maximum on the graph of v
valid approach to find v when $a = 0$ (may be seen on graph) ***(M1)***
eg $\frac{\mathrm{d}v}{\mathrm{d}t} = 0,\ 10e^{-1.7t} - 17te^{-1.7t} = 0,\ t = 0.588$
velocity $= 2.16$ (ms^{-1}) ***A1 N3***

Note: Award ***R1M1A0*** for $(0.588, 216)$ if velocity is not identified as final answer

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Let $f(x) = 2x + 4$ and $g(x) = 7x^2$.

a. Find $f^{-1}(x)$. [3]

b. Find $(f \circ g)(x)$. [2]

c. Find $(f \circ g)(3.5)$. [2]

Markscheme

a. interchanging *x* and *y* (may be seen at any time) ***(M1)***

evidence of correct manipulation ***(A1)***

e.g. $x = 2y + 4$

$f^{-1}(x) = \frac{x-4}{2}$ (accept $y = \frac{x-4}{2}$, $\frac{x-4}{2}$) ***A1*** ***N2***

[3 marks]

b. attempt to form composite (in any order) ***(M1)***

e.g. $f(7x^2)$, $2(7x^2) + 4$, $7(2x + 4)^2$

$(f \circ g)(x) = 14x^2 + 4$ ***A1*** ***N2***

c. correct substitution ***(A1)***

e.g. 7×3.5^2 , $14(3.5)^2 + 4$

$(f \circ g)(3.5) = 175.5$ (accept 176) ***A1*** ***N2***

[2 marks]

Examiners report

a. All parts of this question were well answered by most of the candidates. Some misunderstood part (a) and found the derivative or the reciprocal, indicating they were not familiar with the notation for an inverse function. Occasionally, the composition symbol was mistaken for multiplication. Additionally, some candidates composed in the incorrect order.

b. All parts of this question were well answered by most of the candidates. Some misunderstood part (a) and found the derivative or the reciprocal, indicating they were not familiar with the notation for an inverse function. Occasionally, the composition symbol was mistaken for multiplication. Additionally, some candidates composed in the incorrect order.

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The following diagram shows a circle with centre O and radius r cm.

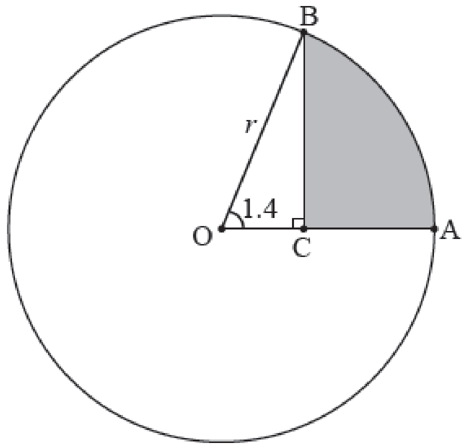


diagram not to scale

Points A and B are on the circumference of the circle and $\mathrm{A\hat{O}B} = 1.4$ radians .

The point C is on [OA] such that $\mathrm{B\hat{C}O} = \frac{\pi}{2}$ radians .

a. Show that $\mathrm{OC} = r\cos 1.4$. [1]

b. The area of the shaded region is $25\ \mathrm{cm}^2$. Find the value of r . [7]

Markscheme

a. use right triangle trigonometry ***A1***

eg $\cos 1.4 = \frac{\mathrm{OC}}{r}$

$\mathrm{OC} = r\cos 1.4$ ***AG*** ***N0***

[1 mark]

b. correct value for BC

eg $\mathrm{BC} = r\sin 1.4$, $\sqrt{r^2 - (r\cos 1.4)^2}$ ***(A1)***

area of $\Delta\mathrm{OBC} = \frac{1}{2}r\sin 1.4 \times r\cos 1.4 \left(= \frac{1}{2}r^2\sin 1.4 \times \cos 1.4\right)$ ***A1***

area of sector $\mathrm{OAB} = \frac{1}{2}r^2 \times 1.4$ ***A1***

attempt to subtract in any order ***(M1)***

eg sector – triangle, $\frac{1}{2}r^2\sin 1.4 \times \cos 1.4 - 0.7r^2$

correct equation ***A1***

eg $0.7r^2 - \frac{1}{2}r\sin 1.4 \times r\cos 1.4 = 25$

attempt to solve ***their*** equation ***(M1)***

eg sketch, writing as quadratic, $\frac{25}{0.616\ldots}$

$r = 6.37$ ***A1*** ***N4***

[7 marks]

Note: Exception to ***FT*** rule. Award ***A1FT*** for a correct ***FT*** answer from a quadratic equation involving two trigonometric functions.

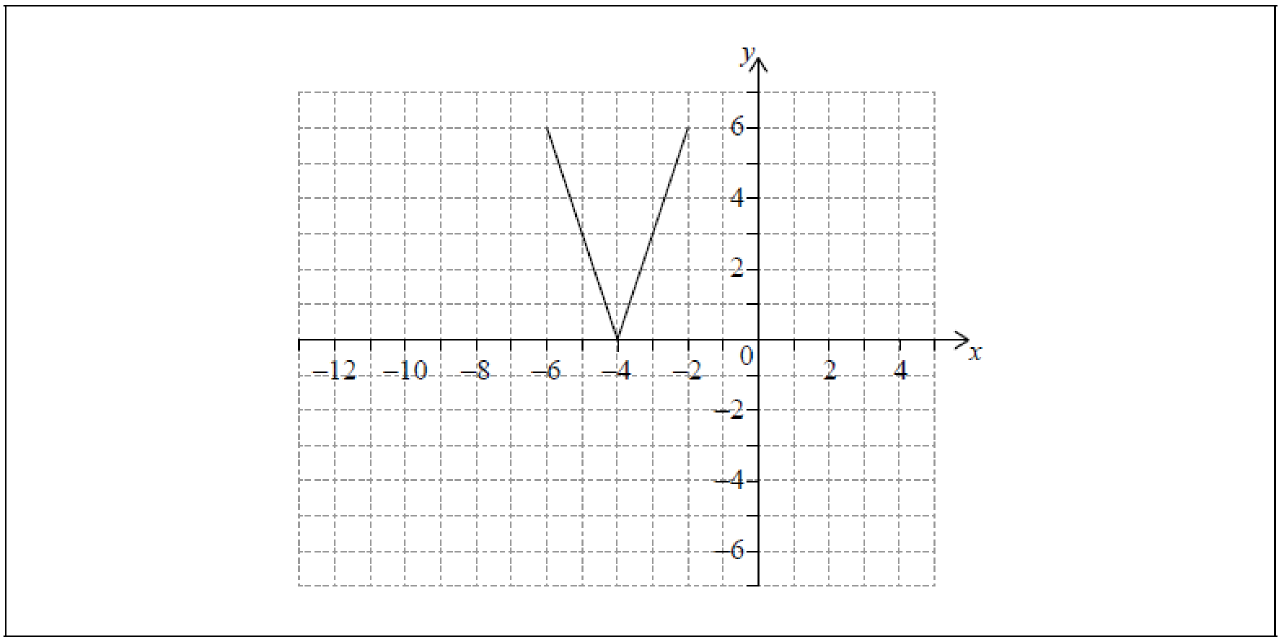
Examiners report

a. As to be expected, candidates found this problem challenging. In part (a), many were able to use right angle trigonometry to find the length of OC.

b. As to be expected, candidates found this problem challenging. Those who used a systematic approach in part (b) were more successful than those whose work was scattered about the page. While a pleasing number of candidates successfully found the area of sector AOB, far fewer were able to find the area of triangle BOC. Candidates who took an analytic approach to solving the resulting equation were generally less successful than those who used their GDC. Candidates who converted the angle to degrees generally were not very successful.

The following diagram shows the graph of a function $y = f(x)$, for $-6 \leqslant x \leqslant -2$.

The points $(-6,\ 6)$ and $(-2,\ 6)$ lie on the graph of f. There is a minimum point at $(-4,\ 0)$.



Let $g(x) = f(x - 5)$.

a. Write down the range of f. [2]

b. On the grid above, sketch the graph of g. [2]

c. Write down the domain of g. [2]

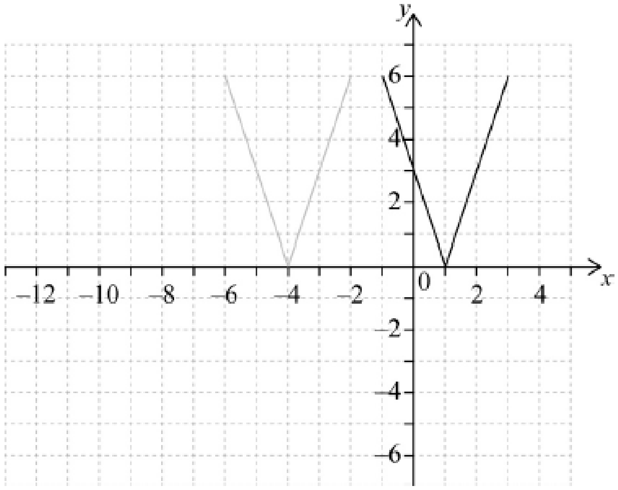
Markscheme

a. correct interval ***A2*** ***N2***

eg $0 \leqslant y \leqslant 6,\ [0,\ 6]$, from 0 to 6

[2 marks]

b.



M1A1 ***N2***

Note: Award ***M1*** for a horizontal shift of the whole shape, 5 units to the left or right and ***A1*** for the correct graph.

[2 marks]

c. correct interval ***A2*** ***N2***

eg $-1 \leqslant x \leqslant 3,\ [-1,\ 3]$, from -1 to 3

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Solve the equation $\mathrm{e}^x = 4\sin x$, for $0 \le x \le 2\pi$.

Markscheme

evidence of appropriate approach ***M1***

e.g. a sketch, writing $\mathrm{e}^x - 4\sin x = 0$

$x = 0.371$, $x = 1.36$ ***A2A2*** ***N2N2***

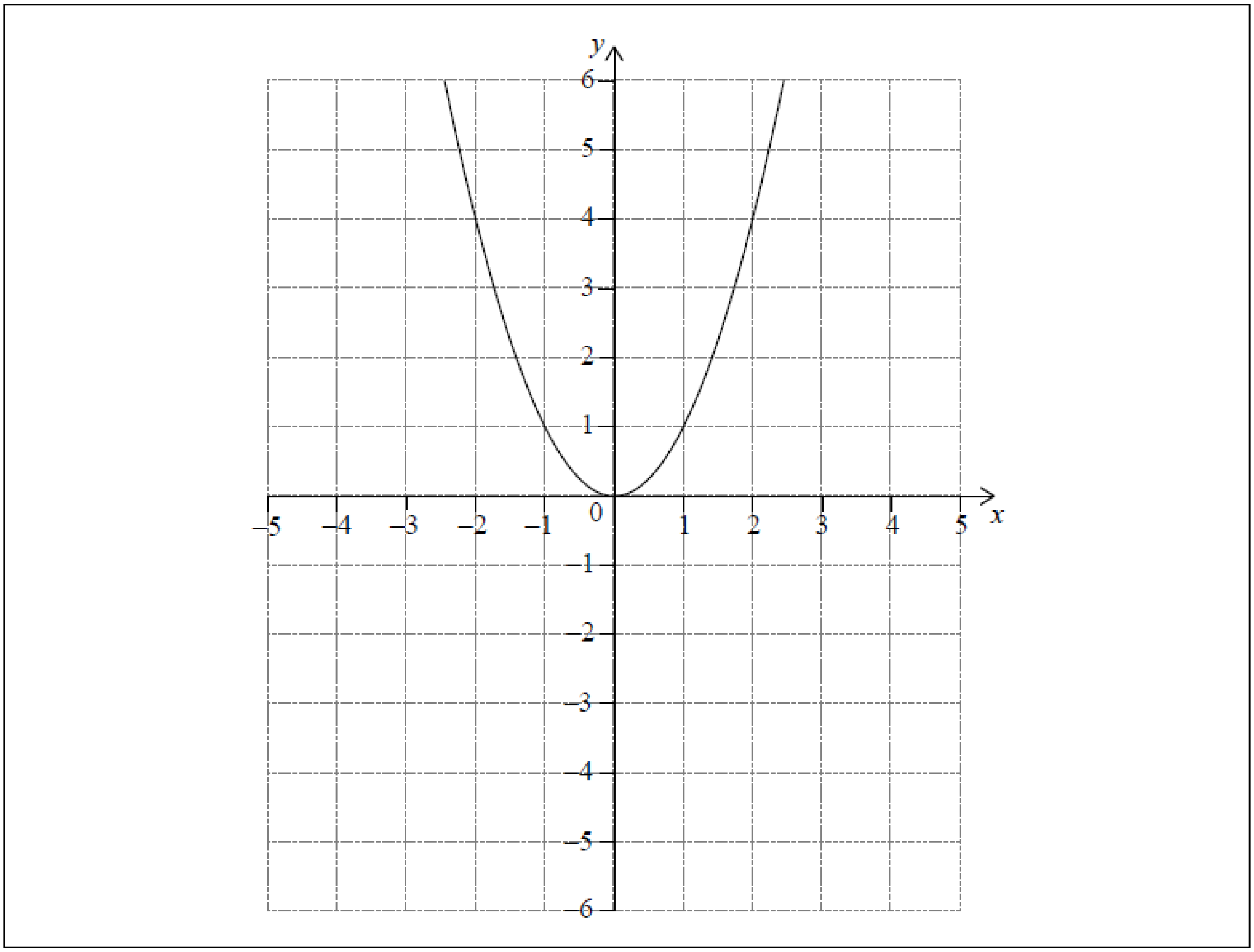
[5 marks]

Examiners report

Although many students started with an analytical approach, many also realized they were not going further and successfully used their GDC to find the intercepts with the *x*-axis if they had set the equation equal to 0, or in other cases, they found the intersection of the two graphs. The better candidates drew a reasonable sketch and found the two values without difficulty. A good number of candidates did not provide a sketch, however, and they had more trouble earning the mark for showing method. Accuracy penalties were relatively common on this question.

Let $g(x) = -(x - 1)^2 + 5$.

Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

a. Write down the coordinates of the vertex of the graph of g. [1]

b. On the grid above, sketch the graph of g for $-2 \le x \le 4$. [3]

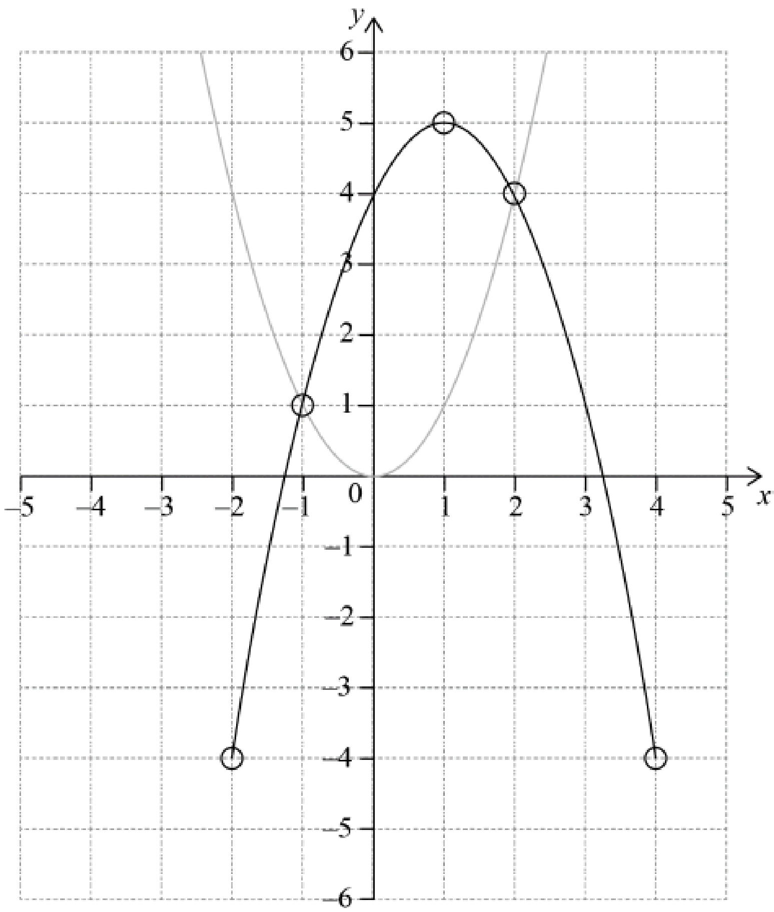
c. Find the area of the region enclosed by the graphs of f and g. [3]

Markscheme

a. (1,5) (exact) ***A1 N1***

[1 mark]

b.



A1A1A1 N3

Note: The shape must be a concave-down parabola.
Only if the shape is correct, award the following for points in circles:
A1 for vertex,
A1 for correct intersection points,
A1 for correct endpoints.

[3 marks]

c. integrating and subtracting functions (in any order) ***(M1)***

eg $\int f - g$

correct substitution of limits or functions (accept missing d*x*, but do not accept any errors, including extra bits) ***(A1)***
eg $\int_{-1}^{2} g - f,\ \int -(x-1)^2 + 5 - x^2$

area = 9 (exact) ***A1 N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

In a geometric series, $u_1 = \frac{1}{81}$ and $u_4 = \frac{1}{3}$.

a. Find the value of r . [3]

b. Find the smallest value of n for which $S_n > 40$. [4]

Markscheme

a. evidence of substituting into formula for nth term of GP ***(M1)***

e.g. $u_4 = \frac{1}{81}r^3$

setting up correct equation $\frac{1}{81}r^3 = \frac{1}{3}$ ***A1***

$r = 3$ ***A1*** ***N2***

[3 marks]

b. **METHOD 1**

setting up an inequality (accept an equation) ***M1***

e.g. $\frac{\frac{1}{81}(3^n-1)}{2} > 40$, $\frac{\frac{1}{81}(1-3^n)}{-2} > 40$, $3^n > 6481$

evidence of solving ***M1***

e.g. graph, taking logs

$n > 7.9888\ldots$ ***(A1)***

$\therefore n = 8$ ***A1*** ***N2***

METHOD 2

if $n = 7$, sum $= 13.49\ldots$; if $n = 8$, sum $= 40.49\ldots$ ***A2***

$n = 8$ (is the smallest value) ***A2*** ***N2***

[4 marks]

Examiners report

a. Part (a) was well done.

b. In part (b) a good number of candidates did not realize that they could use logs to solve the problem, nor did they make good use of their GDCs. Some students did use a trial and error approach to check various values however, in many cases, they only checked one of the "crossover" values. Most candidates had difficulty with notation, opting to set up an equation rather than an inequality.

Let $f(x) = x^3 - 4x + 1$.

a. Expand $(x+h)^3$. [2]

b. Use the formula $f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$. [4]

c. The tangent to the curve of f at the point $\mathrm{P}(1,\ -2)$ is parallel to the tangent at a point Q. Find the coordinates of Q. [4]

d. The graph of f is decreasing for $p < x < q$. Find the value of p and of q. [3]

e. Write down the range of values for the gradient of f . [2]

Markscheme

a. attempt to expand ***(M1)***

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ ***A1*** ***N2***

[2 marks]

b. evidence of substituting $x+h$ ***(M1)***

correct substitution ***A1***

e.g. $f'(x) = \lim_{h\to 0} \frac{(x+h)^3 - 4(x+h) + 1 - (x^3 - 4x + 1)}{h}$

simplifying ***A1***

e.g. $\frac{(x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h + 1 - x^3 + 4x - 1)}{h}$

factoring out h ***A1***

e.g. $\frac{h(3x^2 + 3xh + h^2 - 4)}{h}$

$f'(x) = 3x^2 - 4$ ***AG*** ***N0***

[4 marks]

c. $f'(1) = -1$ ***(A1)***

setting up an appropriate equation ***M1***

e.g. $3x^2 - 4 = -1$

at Q, $x = -1, y = 4$ (Q is $(-1, 4)$) ***A1*** ***A1***

[4 marks]

d. recognizing that *f* is decreasing when $f'(x) < 0$ ***R1***

correct values for p and q (but do not accept $p = 1.15$, $q = -1.15$) ***A1A1*** ***N1N1***

e.g. $p = -1.15$, $q = 1.15$; $\pm\frac{2}{\sqrt{3}}$; an interval such as $-1.15 \le x \le 1.15$

[3 marks]

e. $f'(x) \ge -4$, $y \ge -4$, $[-4, \infty[$ ***A2*** ***N2***

[2 marks]

Examiners report

a. In part (a), the basic expansion was not done well. Rather than use the binomial theorem, many candidates opted to expand by multiplication which resulted in algebraic errors.

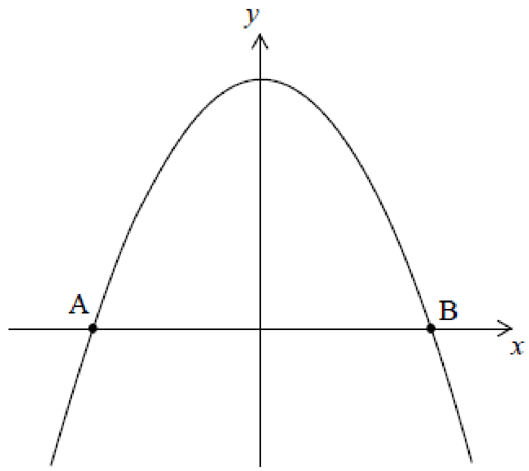
b. In part (b), it was clear that many candidates had difficulty with differentiation from first principles. Those that successfully set the answer up, often got lost in the simplification.

c. Part (c) was poorly done with many candidates assuming that the tangents were horizontal and then incorrectly estimating the maximum of *f* as the required point. Many candidates unnecessarily found the equation of the tangent and could not make any further progress.

d. In part (d) many correct solutions were seen but only a very few earned the reasoning mark.

e. Part (e) was often not attempted and if it was, candidates were not clear on what was expected.

Let $f(x) = 5 - x^2$. Part of the graph of $f$is shown in the following diagram.



The graph crosses the x-axis at the points A and B.

a. Find the x-coordinate of A and of B. [3]

b. The region enclosed by the graph of f and the x-axis is revolved 360° about the x-axis. [3]

Find the volume of the solid formed.

Markscheme

a. recognizing $f(x) = 0$ ***(M1)***

eg $f = 0,\ x^2 = 5$

$x = \pm 2.23606$

$x = \pm\sqrt{5}$ (exact), $x = \pm 2.24$ ***A1A1 N3***

[3 marks]

b. attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\pi \int \left(5 - x^2\right)^2 \mathrm{d}x,\ \pi \int_{-2.24}^{2.24} \left(x^4 - 10x^2 + 25\right),\ 2\pi \int_0^{\sqrt{5}} f^2$

187.328

volume $= 187$ ***A2 N3***

[3 marks]

Examiners report

a. [N/A]

b. [N/A]